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Neutron Scaling Laws from Numerical Experiments

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Abstract

Experimental data of neutron yield Y_n against pinch current I_{pinch} is assembled to produce a more global scaling law than available. From the data a mid-range point is obtained to calibrate the neutron production mechanism of the Lee Model code. This code is then used for numerical experiments on a range of focus devices to derive neutron scaling laws. The results are the following: $Y_n=2x10^{11}I_{pinch}^{4.7}$ and $Y_n=9x10^{9}I_{peak}^{3.9}$. It is felt that the scaling law with respect to I_{pinch} is rigorously obtained by these numerical experiments when compared with that obtained from measured data, which suffers from inadequacies in the measurements of I_{pinch}.

Keywords: Plasma Focus Neutron Scaling Pinch Current Focus modelling Lee Model

Introduction

A major feature of the plasma focus is its fusion neutron yield. Even a simple trolley mounted 3kJ device such as the UNU/ICTP PFF routinely produces[1] a yield of $Y_n=10^8$ neutrons, operating in deuterium. A big machine such as the PF1000 typically produces 10^{11} neutrons per shot[2]. Moreover since the neutrons are produced in a short pulse of the order of 10ns, the rate of neutron production is 10^{16} neutrons/s even for a small machine and can go up to 10^{20} for a large machine.

From a compilation of experimental data over a wide range of energies a scaling law of $Y_n{\sim}I_{pinch}^{3.3}$ was presented by Bernard[3], where I_{pinch} is the current flowing through the dense pinch in the focused plasma. Kies[4] presented another compilation showing $Y_n{\sim}I_{pinch}^{4}$ whilst Herold[5] had results showing $Y_n{\sim}I_{pinch}^{3.2}$. Gribkov has recently[2] suggested that the experimental data can be interpreted with the power law as high as 5 in particular when dealing with the same device.

One significant uncertainly in compiling such a scaling law is the interpretation of I_{pinch}. The current most conveniently measured in most experiments is the total current flowing into the tube (usually measured with a Rogowski coil placed at the collector plate just outside the tube). This total current has a maximum value I_{peak}. If one estimates I_{pinch} from the total current measurement there are two difficulties: 1. it is difficult to determine the point on the current waveform where the plasma has gone into the pinch phase, and 2. even after estimating this point, it still remains to estimate the fraction of total current that in fact flows into the pinch. One way is to use small magnetic coils to probe the pinch region. For small machines this method is not suitable because of the amount of space available and the small size of the pinch so that the probes inevitably interfere with the pinching current sheet. For large machines, results have been obtained[5] but with large errors quoted as 20%. Moreover the shot-to-shot variability of focus performance means that the final presentation of results relies greatly on how the particular research group chooses to present the results. For example the yield may be presented as a range, with some shots considered not representative discarded, and perhaps the biggest values of observed yield also presented. It is quite remarkable that despite all these difficulties there is a consensus of opinion that the index in this power scaling law has the value in the range of 3 to 5.

Compilation of experimental results

In this paper we have combined the laboratory data that we have[1-7], which includes recent results from some smaller machines e.g. Soto's[6] PF400 and the large[2] PF1000 as well as a high performance repetitive device[7], the NX2. This gives a good fit of $Y_n=9\times10^{10}I_{pinch}^{3.8}$. The main reason for this compilation of experimental results is to provide a calibration point for setting the neutron yield mechanism of the Lee Model code, described below. A calibration point is chosen at around the middle of the current range at I_{pinch}=0.5MA, $Y_n=6\times10^9$ neutrons. This point is close to the PF1000's machine parameters with properly adjusted dimensions if it could be fired at 13.5kV.

The results of the compilation are shown in Fig 1.

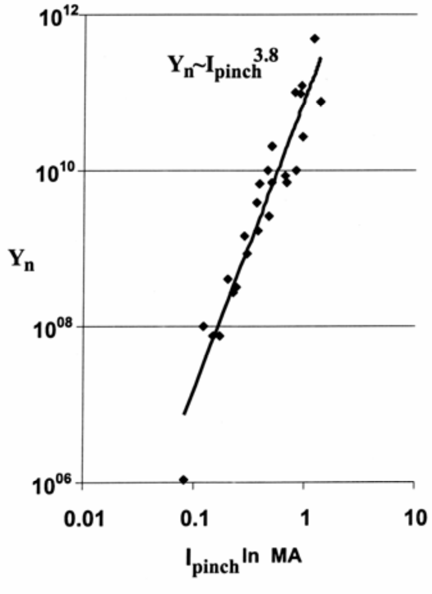


Fig 1. Y_n scaling with I_{pinch} from laboratory data

The Model used for the numerical experiments

The Lee Model has been widely used to simulate axial and radial phase dynamics, temperatures and thermodynamic properties and radiation yields. To realistically simulate any plasma focus all that is needed is a measured current trace of that plasma focus. Recently the model code[8] has been extended to include a phenomenological beam-target mechanism based partially on that proposed by Gribkov[2].

The main mechanism producing the neutrons is a beam of fast deuteron ions interacting with the hot dense plasma of the focus pinch column. The fast ion beam is produced by diode action in a thin layer close to the anode with plasma disruptions generating the necessary high voltages. This mechanism, described in some details in a recent paper[9], results in the following expression used for the model code:

$$Y_{b\text{-}t} = \text{calibration constant x } \mathbf{n_i}\, I_{pinch}^{2}\, z_p^{2}(\ln(b/r_p))\sigma/V_{max}^{0.5}$$

where I_{pinch} is the current at the start of the slow compression phase, r_p and z_p are the pinch radius and pinch length at the end of the slow compression phase, V_{max} is the maximum value attained by the inductively induced voltage, σ is the D-D fusion cross section (n branch)[10] corresponding to the beam ion energy **and n_i is the pinch ion density**. The D-D cross section σ is obtained by using beam energy equal to 3 times V_{max}, to conform to experimental observations.

Scaling Laws derived from the numerical experiments

This paper applies the code to several machines including the PF400, UNU/ICTP PFF, the NX2 and Poseidon. The PF1000 which has a current curve published at 27kV and Y_n published at 35kV provided an important point. Moreover using parameters for the PF1000 established at 27 kV and 35 kV, additional points were taken at different voltages ranging from 13.5kV upwards to 40kV.

These machines were chosen because each has a published current trace and hence the current curve computed by the model code could be fitted to the measured current trace. Once this fitting is done our experience is that the other computed properties including dynamics, energy distributions and radiation are all realistic. This gives confidence that the computed Y_n for each case is also realistic. Moreover since each chosen machine also has measured Y_n corresponding to the current trace, the computed Y_n could also be compared with the measured to ensure that the computed results are not incompatible with the measured values.

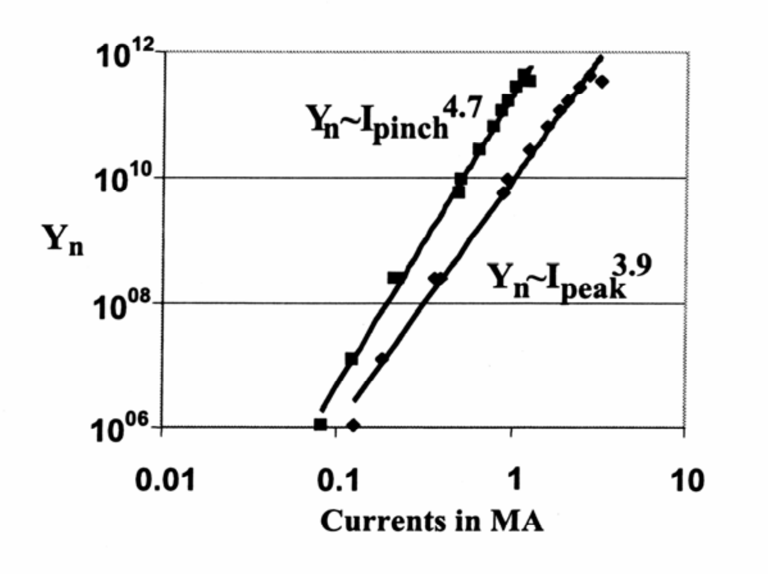
The results are shown in Table 1 and Fig 2.

In Table 1, corresponding to each laboratory device, the operating voltage V_o and pressure P_o are typical of the device, as is the capacitance C_o. It was found that the static inductance L_o usually needed to be adjusted from the value provided by the laboratory. This is because the value provided could be for short circuit conditions, or an estimate including the input flanges and hence that value may not be sufficiently close to L_o. The dimensions b (outer radius), a (anode radius) and z_o (anode length) are also the typical dimensions for the specific device. The speed factor[11] S is also included. All devices except Poseidon have typical S values. Poseidon is the exceptional high speed device in this respect. The minimum pinch radius is also tabulated as $k_{min}=r_p/a$. It is noted that this parameter increases from 0.14 for the smaller machines towards 0.2 for the biggest machines. The ratio I_{pinch}/I_{peak} is also tabulated showing a trend of decreasing from 0.65 for small machines to 0.4 for the biggest machines.

Table 1. Computed values of I_{peak}, I_{pinch} and Y_n for a range of Plasma Focus Machines

Machine	V_o (kV)	P_o (torr)	L_o (nH)	C_o (μF)	b (cm)	a (cm)	Z_o (cm)	I_{peak} (MA)	I_{pinch} (MA)	S	Y_n	k_{min}	I_{pinch}/I_{peak}
PF400	28	6.6	40	0.95	1.55	0.60	1.7	0.126	0.082	82	1.1 x 10^{06}	0.14	0.65
UNU	15	4	110	30	3.2	0.95	16	0.182	0.123	96	1.2 x 10^{07}	0.14	0.68
NX2 T	15	5	20	28	5	2	7	0.386	0.225	86	2.5 x 10^{08}	0.16	0.58
Calibration	16	5	24	308	7	4	30	0.889	0.496	99	5.6 x 10^{09}	0.17	0.56
NX2 T-2	12.5	10.6	19	28	3.8	1.55	4	0.357	0.211	71	2.4 x 10^{08}	0.16	0.59
PF1000	13.5	3.5	33	1332	8.00	5.78	60	0.924	0.507	89	9.6 x 10^{09}	0.17	0.55
	18	3.5	33	1332	10.67	7.70	60	1.231	0.636	89	2.9 x 10^{10}	0.18	0.52
	23	3.5	33	1332	13.63	9.84	60	1.574	0.766	89	6.8 x 10^{10}	0.19	0.49
	27	3.5	33	1332	16	11.60	60	1.847	0.862	89	1.2 x 10^{11}	0.19	0.47
	30	3.5	33	1332	17.77	12.80	60	2.049	0.929	89	1.6 x 10^{11}	0.20	0.45
	35	3.5	33	1332	20.74	15.00	60	2.399	1.037	89	2.7 x 10^{11}	0.20	0.43
	40	3.5	33	1332	23.70	17.10	60	2.736	1.137	89	4.1 x 10^{11}	0.21	0.42
Poseidon	60	3.8	18	156	9.50	6.55	30	3.200	1.260	251	3.3 x 10^{11}	0.20	0.39

Fig 2. Y_n scaling with I_{pinch} and I_{peak} from numerical experiments



The resultant data with improved optimization yield more up to date scaling laws:$Y_n \sim I_{pinch}^{4.7}$ and $Y_n \sim I_{peak}^{3.9}$. It is necessary to emphasize again that the I_{pinch} may be considered to be computed rigorously especially for those cases where an experimental current curve is available. Once the computed current curve is fitted accurately to the experimental current curve, the resultant pinch position is pinpointed as well as the fraction of current going into the pinch.

This is in contrast to the laboratory data where I_{pinch} is usually only estimated and if measured is subject to large errors. A study of the data suggests that in most cases I_{pinch} is overestimated by experimentalists. With all these considerations it would appear that the scaling laws arising from the code are not inconsistent with experimental observations and may complement the more conventionally compiled scaling laws to provide comprehensive database for experiments.

Conclusion

Neutron scaling laws have been derived from computation using the Lee Model code. These are: $Y_n \sim I_{pinch}^{4.7}$ and $Y_n \sim I_{peak}^{3.9}$. In these numerical experiments I_{pinch} is rigorously computed whereas in compilation of laboratory results I_{pinch} is usually just guessed at or at best estimated. These numerically derived scaling laws are not inconsistent with compilation from laboratory experiments. The numerically derived scaling law against I_{pinch} has an index of 4.7 which is higher than the usually accepted scaling law with index of 3.2 to 4. The indications are that the numerically derived scaling laws being more rigorous and consistent in derivation may actually be more realistic and more reliable for use in interpreting, designing or planning experiments.

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Erratum

This version of the paper contains two additions to the published paper on pg 3. The paragraph containing the additions is reproduced here in parenthesis, with the additions highlighted in bold red:

$$"Y_{b\text{-}t} = \text{calibration constant x } \mathbf{n_i}\, I_{pinch}^{2}\, z_p^{2}(\ln(b/r_p))\sigma/V_{max}^{0.5}$$

where I_{pinch} is the current at the start of the slow compression phase, r_p and z_p are the pinch radius and pinch length at the end of the slow compression phase, V_{max} is the maximum value attained by the inductively induced voltage and σ is the D-D fusion cross section (n branch)[10] corresponding to the beam ion energy **and n_i is the pinch ion density**. The D-D cross section σ is obtained by using beam energy equal to 3 times V_{max}, to conform to experimental observations."