Neutron yield saturation in plasma focus: A fundamental cause

S. Lee[a)]
INTI University College, 71800 Nilai, Malaysia; Institute for Plasma Focus Studies, 32 Oakpark Dr, Chadstone VIC3148, Australia; and National Institute of Education, Nanyang Technological University, Singapore 637616, Singapore

(Received 28 July 2009; accepted 21 September 2009; published online 15 October 2009)

Plasma focus research in the direction of fusion energy faces the limitation of observed neutron saturation; the neutron yield Y_n falls away from $Y_n \sim E_0^2$, the scaling deteriorating as storage energy E_0 increases toward 1 MJ. Numerical experiments confirm that $Y_n \sim E_0^2$ applies at low energies and drops to $Y_n \sim E_0^{0.8}$ toward 25 MJ; deteriorating already at several hundred kilojoules. We point out that the cause is the dynamic resistance of the axial phase that is constant for all plasma foci. This dynamic resistance dominates the circuit as capacitor bank surge impedance becomes insignificant at large E_0, causing current, hence neutron "saturation." © *2009 American Institute of Physics*. [doi:10.1063/1.3246159]

It was observed early in plasma focus research[1] that neutron yield $Y_n \sim E_0^2$ where E_0 is the capacitor storage energy. Such scaling gave hopes of possible development as a fusion energy source. Devices were scaled up to higher E_0. It was then observed that the scaling deteriorated, with Y_n not increasing as much as suggested by the E_0^2 scaling. In fact some experiments were interpreted as evidence of a neutron saturation effect[2,3] as E_0 approached several hundreds of kilojoules. As recently as 2006, Kraus[4] and Scholz[5] (November 2007) have questioned whether the neutron saturation was due to a fundamental cause or to avoidable machine effects such as incorrect formation of plasma current sheath arising from impurities or sheath instabilities.[3] We should note here that the region of discussion (several hundreds of kilojoules approaching the megajoules region) is in contrast to the much higher energy region discussed by Schmidt[6] at which there might be expected to be a decrease in the role of beam target fusion processes.[3]

Recent extensive numerical experiments[7,8] also showed that whereas at energies up to tens of kilojoules the $Y_n \sim E_0^2$ scaling held, deterioration of this scaling became apparent above the low hundreds of kilojoules. This deteriorating trend worsened and tended toward $Y_n \sim E_0^{0.8}$ at tens of megajoules. The results of these numerical experiments are summarized in Fig. 1 with the solid line representing results from numerical experiments. Experimental results from 0.4 kJ to megajoules, compiled from several available published sources[3,9–14] are also included as squares in the same figure. The combined experimental and numerical experimental results[15] appear to have general agreement particularly with regards to the $Y_n \sim E_0^2$ at energies up to 100 kJ, and the deterioration of the scaling from low hundreds of kilojoules to the 1 MJ level. It is proposed here that the global data of Fig. 1 suggest that the apparently observed neutron saturation effect is overall not in significant variance with the deterioration of the scaling shown by the numerical experiments.

We wish now to provide a simple yet compelling analysis of the cause of this neutron saturation. In Fig. 2 is shown a schematic of the plasma dynamics in the axial phase of the Mather-type plasma focus.

a)Electronic mail: leesing@optusnet.com.au.

We consider the simplest representation in which the current sheet is shown to go from the anode to the cathode perpendicularly. Observation shows that there is actually a canting of the current sheet[16] and also that only a fraction (typically 0.7) of the total current participates in driving the current sheet. These points are accounted for in the modeling[17–22] by model parameters f_m and f_c. For the moment we do not consider these two effects. The outer cathode radius is shown as b, inner anode radius as a and the moving current sheet is shown at position z in the axial phase.

By surveying published results of all Mather-type experiments we find that all deuterium plasma focus devices operate at practically the same speeds[23] and are characterized by a constancy of energy density (per unit mass) over the whole range of devices from the smallest subkilojoule to the largest megajoule devices. The time varying tube inductance is $L=(\mu/2\pi)\ln(c)z$, where $c=b/a$ and μ is the permeability of free space. The rate of change in inductance is $dL/dt=2\times10^{-7}(\ln c)\, dz/dt$ in SI units. Typically on switching, as the capacitor discharges, the current rises toward its peak value, the current sheet is accelerated, quickly reaching nearly its peak speed, and continues accelerating slightly toward its peak speed at the end of the axial phase. Thus for most of its

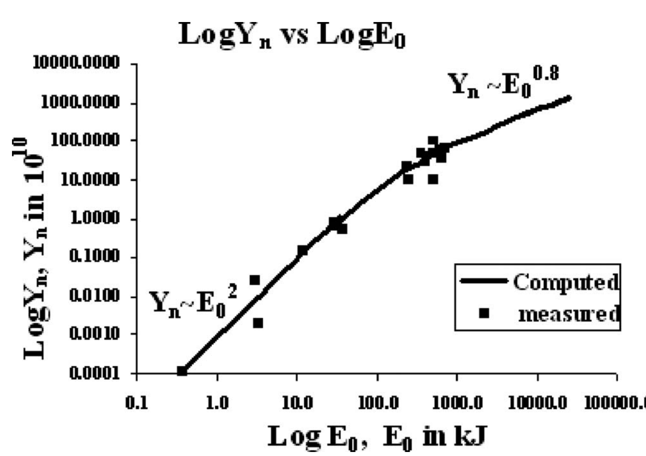

FIG. 1. Illustrating Y_n scaling deterioration observed in numerical experiments from 0.4 kJ to 25 MJ (solid line) using the Lee model code, compared to measurements compiled from publications (squares) of various machines from 0.4 kJ to 1 MJ.

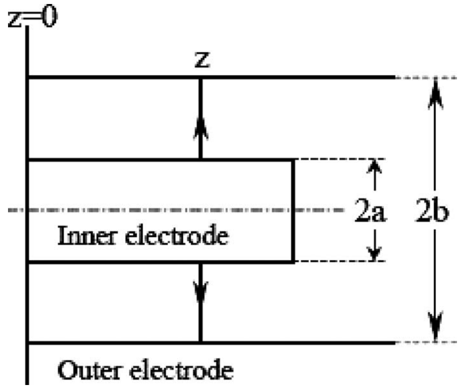

FIG. 2. Plasma focus schematic showing axial phase only.

axial distance the current sheet is traveling at a speed close to the end-axial speed. In deuterium the end-axial speed is observed to be about 10 cm/μs over the whole range of devices. This fixes the rate of change in inductance dL/dt as 1.4×10^{-2} H/s for all the devices, if we take the radius ratio $c=b/a=2$. This value of dL/dt changes by at most a factor of 2, taking into account the variation in c from low values of 1.4 (generally for larger machines) to 4 (generally for smaller machines). This typical dL/dt may also be expressed as 14 mΩ.

We need now to inquire into the nature of the change in the inductance $L(t)$. Consider instantaneous power P delivered to $L(t)$ by a change in $L(t)$.

Induced voltage:

$$V = d(LI)/dt = I(dL/dt) + L(dI/dt). \tag{1}$$

Hence instantaneous power into $L(t)$,

$$P = VI = I^2(dL/dt) + LI(dI/dt). \tag{2}$$

Next, consider instantaneous power associated with the inductive energy $(1/2LI^2)$

$$P_L = d\left(\tfrac{1}{2}LI^2\right)/dt = \tfrac{1}{2}I^2(dL/dt) + LI(dI/dt). \tag{3}$$

We note that P_L of Eq. (3) is not the same as P of Eq. (2).

The difference$=P-P_L=\left(\frac{1}{2}\right)(dL/dt)I^2$ is not associated with the inductive energy stored in L. We conclude that whenever $L(t)$ changes with time, the instantaneous power delivered to $L(t)$ has a component that is not inductive. Hence this component of power $\left(\frac{1}{2}\right)(dL/dt)I^2$ must be resistive in nature; and the quantity $\left(\frac{1}{2}\right)(dL/dt)$ is identified as a resistance due to the motion associated with dL/dt, which we call the dynamic resistance.[15] Note that this is a general result and is independent of the actual processes involved. In the case of the plasma focus axial phase, the motion of the current sheet imparts power to the shock wave structure with consequential shock heating, Joule heating, ionization, radiation etc. The total power imparted at any instant is just the amount $\left(\frac{1}{2}\right)(dL/dt)I^2$, with this amount powering all consequential processes. We denote the dynamic resistance of the axial phase as DR_0.

We have thus identified for the axial phase of the plasma focus a typical dynamic resistance of 7 mΩ due to the motion of the current sheet at 10 cm/μs. It should be noted here that similar ideas of the role of dL/dt as a resistance was discussed by Bernard *et al.*[3] In that work the effect of dL/dt was discussed only for the radial phase. In our opinion the more important phase for the purpose of neutron saturation is actually the axial phase for the Mather-type plasma focus.

TABLE I. Discharge characteristics of equivalent plasma focus circuit, illustrating the saturation of I_{peak} with increase of E_0 to very large values. The last column presents results using circuit (LCR) computation, with a fixed resistance load of 7 mΩ, simulating the effect of the DR_0 and a stray resistance of value $0.1Z_0$.

E_0 (kJ)	C_0 (μF)	Z_0 (mΩ)	DR_0 (mΩ)	Z_{total} (mΩ)	$I_{peak}=V_0/Z_{total}$ (kA)	I_{peak}, LCR (kA)
0.45	1	173	7	197	152	156
4.5	10	55	7	67	447	464
45	100	17	7	26	1156	1234
135	300	10	7	18	1676	1819
450	1000	5.5	7	12.9	2321	2554
1080	2400	3.5	7	10.8	2781	3070
4500	10 000	1.7	7	8.8	3407	3722
45 000	100 000	0.55	7	7.6	4209	4250

We now resolve the problem into its most basic form as follows. We have a generator (the capacitor charged to 30 kV), with an impedance of $Z_0=(L_0/C_0)^{0.5}$ driving a load with a near constant resistance of 7 mΩ. We also assign a value for stray resistance of $0.1Z_0$. This situation may be shown in Table I where L_0 is given a typical value of 30 nH. We also include in the last column the results from a circuit (LCR) computation, discharging the capacitor with initial voltage 30 kV into a fixed resistance load of 7 mΩ, simulating the effect of the DR_0 and a stray resistance of value $0.1Z_0$.

Plotting the peak current as a function of E_0 we obtain Fig. 3, which shows the tendency of the peak current toward saturation as E_0 reaches large values; the deterioration of the curve becoming apparent at the several hundred kilojoule level. This is the case for $I_{peak}=V_0/Z_{tot}$ and also for the LCR discharge with simulated value of the DR_0. In both cases it is seen clearly that a capacitor bank of voltage V_0 discharging into a constant resistance such as DR_0 will have a peak current I_{peak} approaching an asymptotic value of $I_{peak}=V_0/DR_0$ when the bank capacitance C_0 is increased to such large values that the value of $Z_0=(L_0/C_0)^{0.5}\ll DR_0$. Thus DR_0 causes current saturation.

Recent numerical experiments[7,8] have shown agreement with accumulated laboratory data in deriving the relationship between Y_n and I_{peak} and I_{pinch} as follows:

$$Y_n \sim I_{pinch}^{4.5},$$

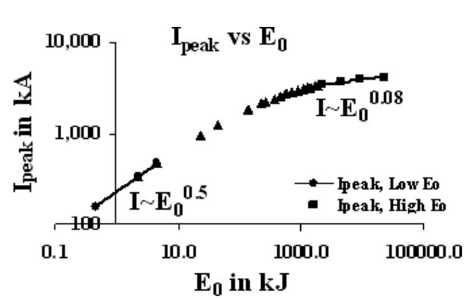

FIG. 3. I_{peak} vs E_0 on log-log scale, illustrating I_{peak} saturation at large E_0.

$$Y_n \sim I_{\text{peak}}^{3.8}.$$

Hence saturation of I_{peak} will lead to saturation of Y_n.

At this point we note that if we consider that only 0.7 of the total current takes part in driving the current sheet, as typically agreed upon from experimental observations, then there is a correction factor which reduces the axial dynamic resistance by some 40%. That would raise the asymptotic value of the current by some 40%, nevertheless there would still be saturation.

In this paper we have shown that current saturation is inevitable as E_0 is increased to very large values by an increase in C_0, simply due to the dominance of the axial phase dynamic resistance. This makes the total circuit impedance tend toward an asymptotic value which approaches the dynamic resistance at infinite values of E_0. The saturation of current inevitably leads to a saturation of neutron yield. Thus the apparently observed neutron "saturation" which is more accurately represented as a neutron scaling deterioration is inevitable because of the dynamic resistance. In line with current plasma focus terminology we will continue to refer to this scaling deterioration as saturation. The above analysis applies to the Mather-type plasma focus. The Filippov-type plasma focus does not have a clearly defined axial phase. Instead it has a liftoff phase and an extended prepinch radial phase which determine the value of I_{peak}. During these phases the inductance of the Filippov discharge is changing, and the changing $L(t)$ will develop a dynamic resistance which will also have the same current saturation effect as the Filippov bank capacitance becomes big enough.

Moreover the saturation as observed in presently available data is due also to the fact that all tabulated machines operate in a narrow range of voltages of 15–50 kV. Only the SPEED machines, most notably SPEED II (Ref. 24) operated at low hundreds of kilovolts. No extensive data have been published from the SPEED machines. Moreover SPEED II, using Marx technology, has a large bank surge impedance of 50 mΩ, which itself would limit the current. If we operate a range of such high voltage machines at a fixed high voltage, say 300 kV, with ever larger E_0 until the surge impedance becomes negligible due to the very large value of C_0. then the saturation effect would still be there, but the level of saturation would be proportional to the voltage. In this way we can go far above presently observed levels of neutron saturation; moving the research, as it were into presently beyond-saturation regimes.

[1] H. Rapp, Phys. Lett. A **43**, 420 (1973).
[2] H. Herold, A. Jerzykiewicz, M. Sadowski, and H. Schmidt, Nucl. Fusion **29**, 1255 (1989).
[3] A. Bernard, H. Bruzzone, P. Choi, H. Chuaqui, V. Gribkov, J. Herrera, K. Hirano, A. Krejci, S. Lee, and C. Luo, J Moscow Physical Society **8**, 93 (1998).
[4] V. I. Krauz, Plasma Phys. Controlled Fusion **48**, B221 (2006).
[5] M. Scholz, Proceedings of the ICDMP Meeting, Warsaw, Poland, November 2007 (unpublished).
[6] H. Schmidt, Proceedings of Fifth International Workshop on Plasma Focus and *Z*-pinch Research, Toledo 1987 (unpublished), p. 65.
[7] S. Lee and S. H. Saw, J. Fusion Energy **27**, 292 (2008).
[8] S. Lee, Plasma Phys. Controlled Fusion **50**, 105005 (2008).
[9] W. Kies, in *Laser and Plasma Technology*, edited by S. Lee, B. C. Tan, C. S. Wong, A. C. Chew, K. S. Low, H. Ahmad, and Y. H. Chen (World Scientific, Singapore, 1988), pp. 86–137.
[10] H. Herold, in *Laser and Plasma Technology*, edited by C. S. Wong, S. Lee, B. C. Tan, A. C. Chew, K. S. Low, and S. P. Moo (World Scientific, Singapore, 1990), pp. 21–45.
[11] S. Lee, T. Y. Tou, S. P. Moo, M. A. Eissa, A. V. Gholap, K. H. Kwek, S. Mulyodrono, A. J. Smith, S. Suryadi, W. Usada, and M. Zakaullah, Am. J. Phys. **56**, 62 (1988).
[12] L. Soto, P. Silva, J. Moreno, G. Silvester, M. Zambra, C. Pavez, L. Altamirano, H. Bruzzone, M. Barbaglia, Y. Sidelnikov, and W. Kies, Braz. J. Phys. **34**, 1814 (2004).
[13] A. Patran, R. S. Rawat, J. M. Koh, S. V. Springham, T. L. Tan, P. Lee, and S. Lee, Proceedings of the 31st EPS Conference on Plasma Physics, London, 2004 (unpublished), Vol. 28G, p. 4.213.
[14] S. V. Springham, S. Lee, and M. S. Rafique, Plasma Phys. Controlled Fusion **42**, 1023 (2000).
[15] www.plasmafocus.net/IPFS/Papers/keynoteaddressIWPDA09.doc
[16] S. P. Chow, S. Lee, and B. C. Tan, J. Plasma Phys. **8**, 21 (1972).
[17] S. Lee, Radiative Dense Plasma Focus Computation Package: RADPF, www.intimal.edu.my/school/fas/UFLF/; www.plasmafocus.net/IPFS/modelpackage/File1RADPF.htm.
[18] S. Lee and S. H. Saw, Appl. Phys. Lett. **92**, 021503 (2008).
[19] S. Lee, P. Lee, S. H. Saw, and R. S. Rawat, Plasma Phys. Controlled Fusion **50**, 065012 (2008).
[20] S. Lee, S. H. Saw, P. C. K. Lee, R. S. Rawat, and H. Schmidt, Appl. Phys. Lett. **92**, 111501 (2008).
[21] S. H. Saw, P. Lee, R. S. Rawat, and S. Lee, IEEE Trans. Plasma Sci. **37**, 1276 (2009).
[22] S. Lee, S. H. Saw, L. Soto, S. V. Springham, and S. P. Moo, Plasma Phys. Controlled Fusion **51**, 075006 (2009).
[23] S. Lee and A. Serban, IEEE Trans. Plasma Sci. **24**, 1101 (1996).
[24] G. Decker, W. Kies, R. Nadolny, P. Röwekamp, F. Schmitz, G. Ziethen, K. N. Koshelev, and Yu. V. Sidelnikov, Plasma Sources Sci. Technol. **5**, 112 (1996).