

# **Exploring the Plasma Focus-From Electrodynamics to Radiative Collapse**

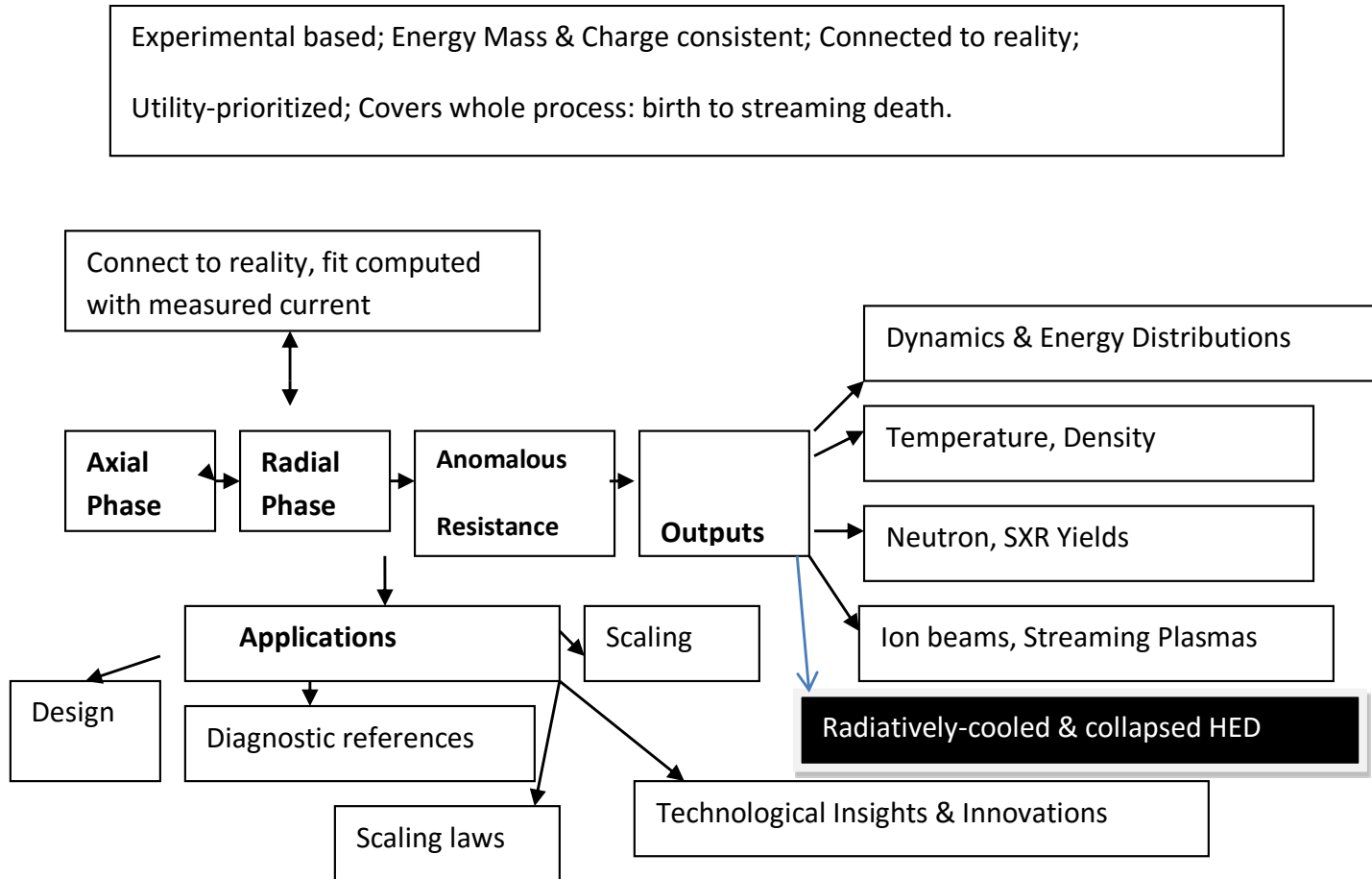
S Lee<sup>1.2.3</sup> and S H Saw<sup>1,2</sup>

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Computations suggest radiative collapse even in small PF devices, opening a new frontier in our multi-faceted numerical journey. This review starts from the **basics**, then touch on **insights** distilled by our efforts, and proceeds to the barely glimpsed world of **radiatively-collapsed high energy density (HED)** in the PF. A schematic of the talk is summarised in the figure below.



# Range of Plasma Focus machines



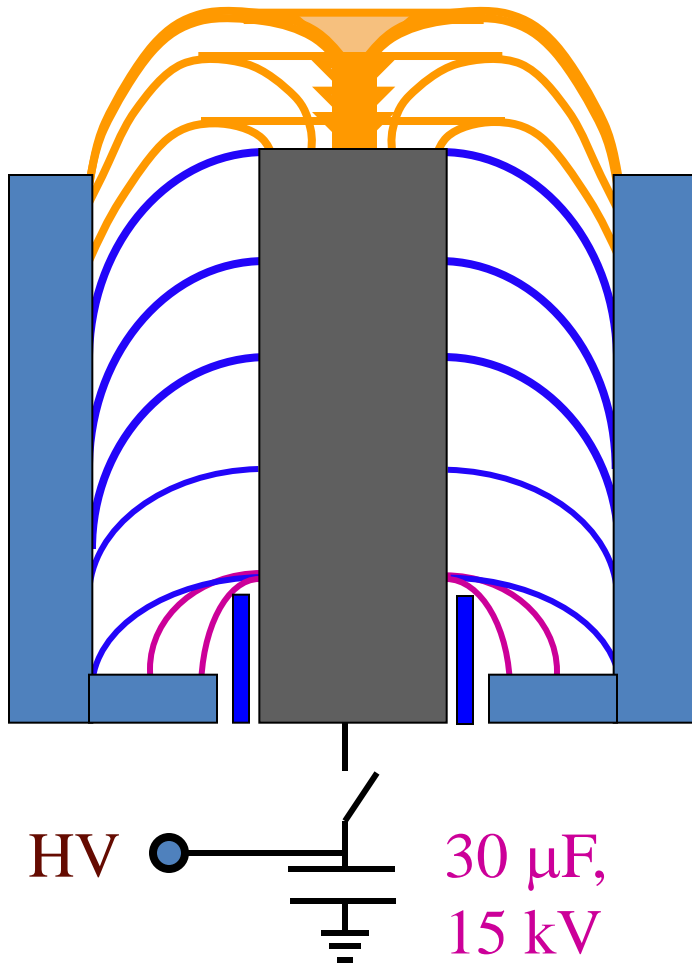
**3 kJ machine**



PF1000 40kV 1332 $\mu$ F 9nH 1.1MJ  $I_0=15$ MA

**1000 kJ machine**

# *Plasma Dynamics in Plasma Focus*

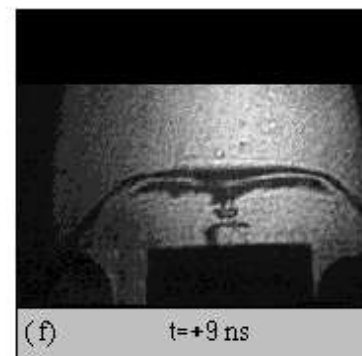
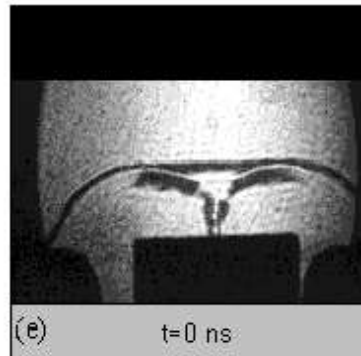
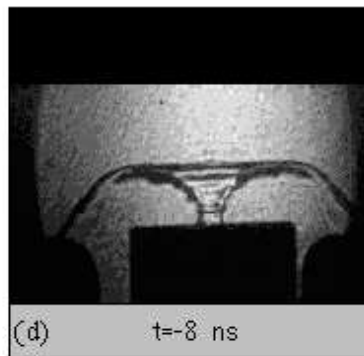
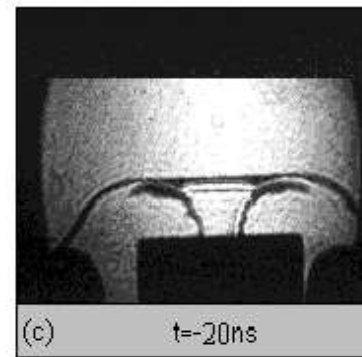
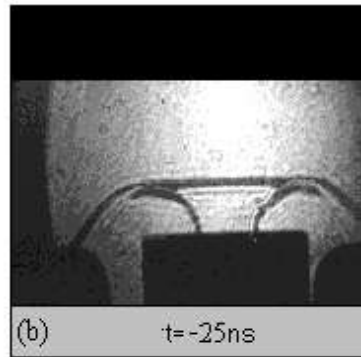
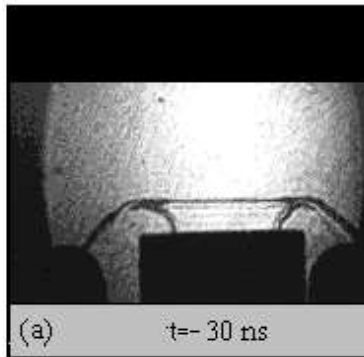


*Radial Phase*

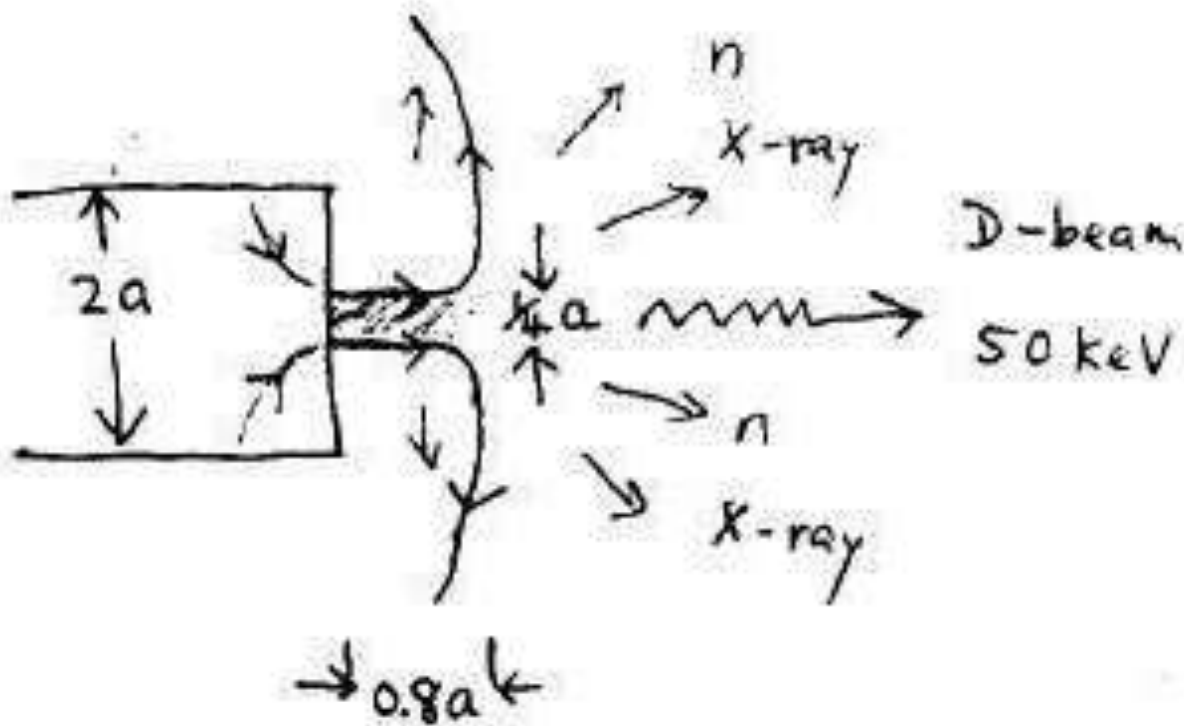
*Axial Acceleration Phase*

*Inverse Pinch Phase*

# Radial Compression (Pinch) Phase of the Plasma Focus



# Emissions from the PF Pinch region



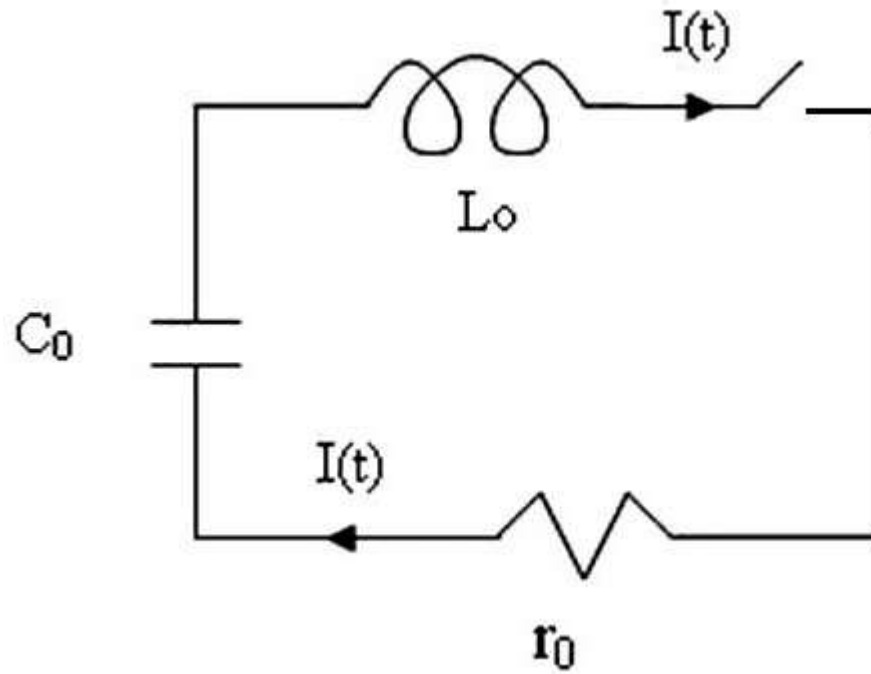
Mach500 Plasma stream  
Mach20 anode material jet

# **Plasma Focus- Numerical Experiments**

## **Parameters of experiments**

- **Capacitor bank parameters**
- **Tube parameters**
- **Operational parameters**
- **Model parameters**

# $L_0$ - $C_0$ - $r_0$ Discharge



# Capacitor bank discharged into short-circuit

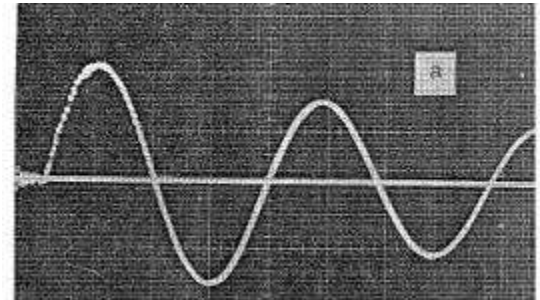
- **Capacitor Discharge**

**Bank parameters:  $L_0, C_0, r_0$       $V_0$**

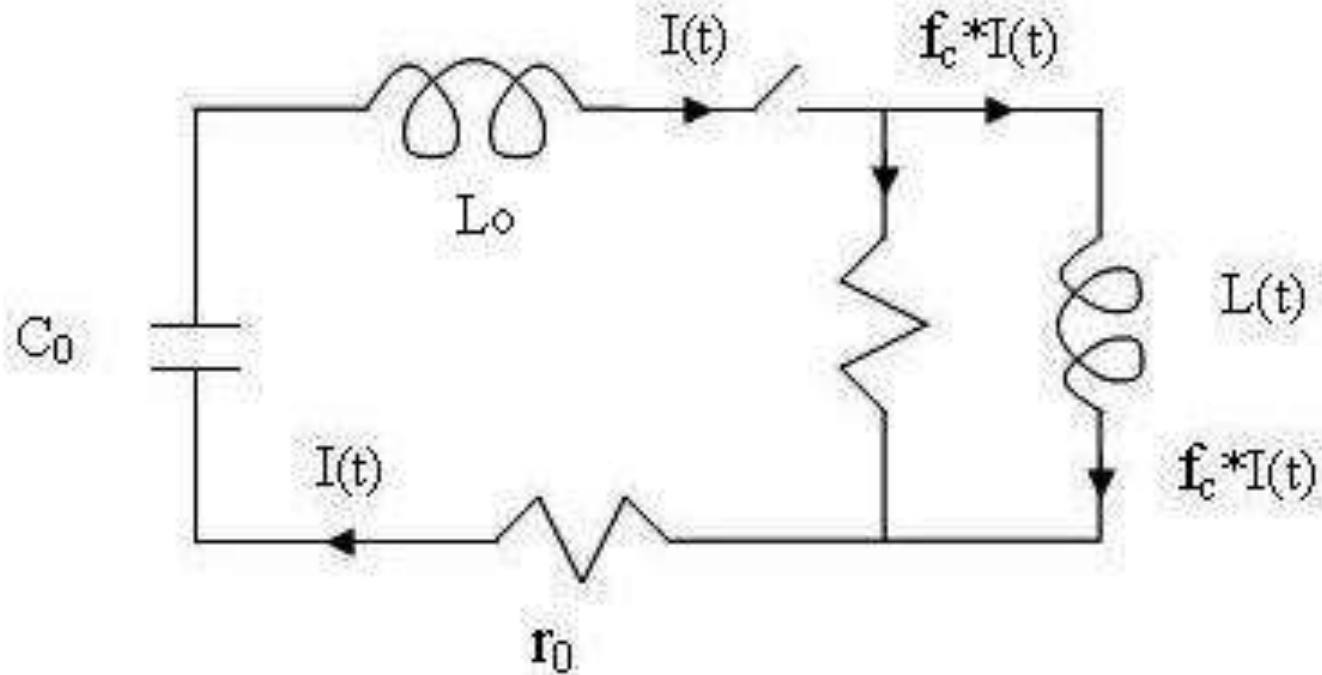
Discharge characteristics:

$L_0$ - $C_0$ - $r_0$  circuit, generates

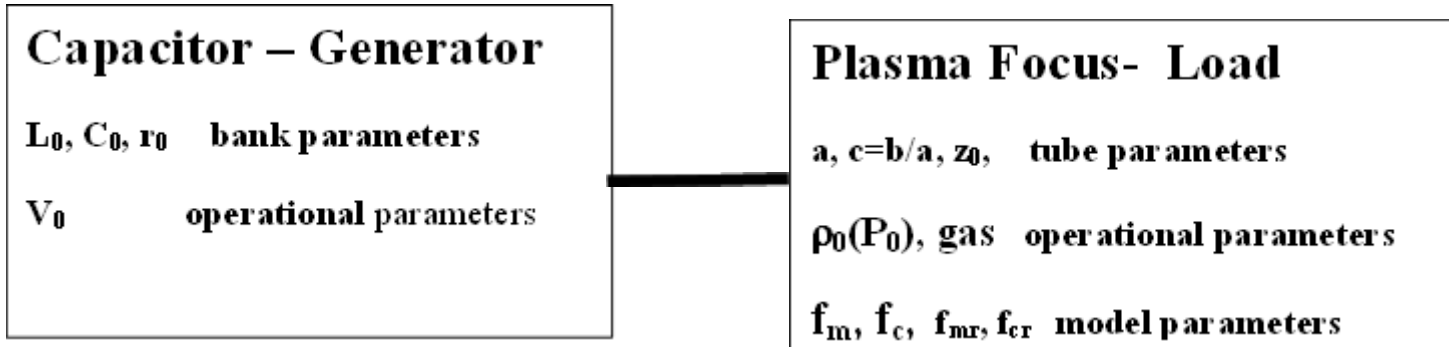
- Damped sinusoidal current
- $T=2\pi(L_0C_0)^{0.5}$
- $I_0=V_0/(L_0/C_0)^{0.5}$



# Plasma focus circuit



# Capacitor bank discharged into PF tube



Damped sinusoidal current (distorted)

$$T = 2\pi(L_0 C_0)^{0.5} \quad \text{increased, distorted}$$

$$I_0 = V_0 / (L_0 / C_0)^{0.5} \quad \text{decreased,}$$

$$t_{\text{rise}} = (1/4)T$$

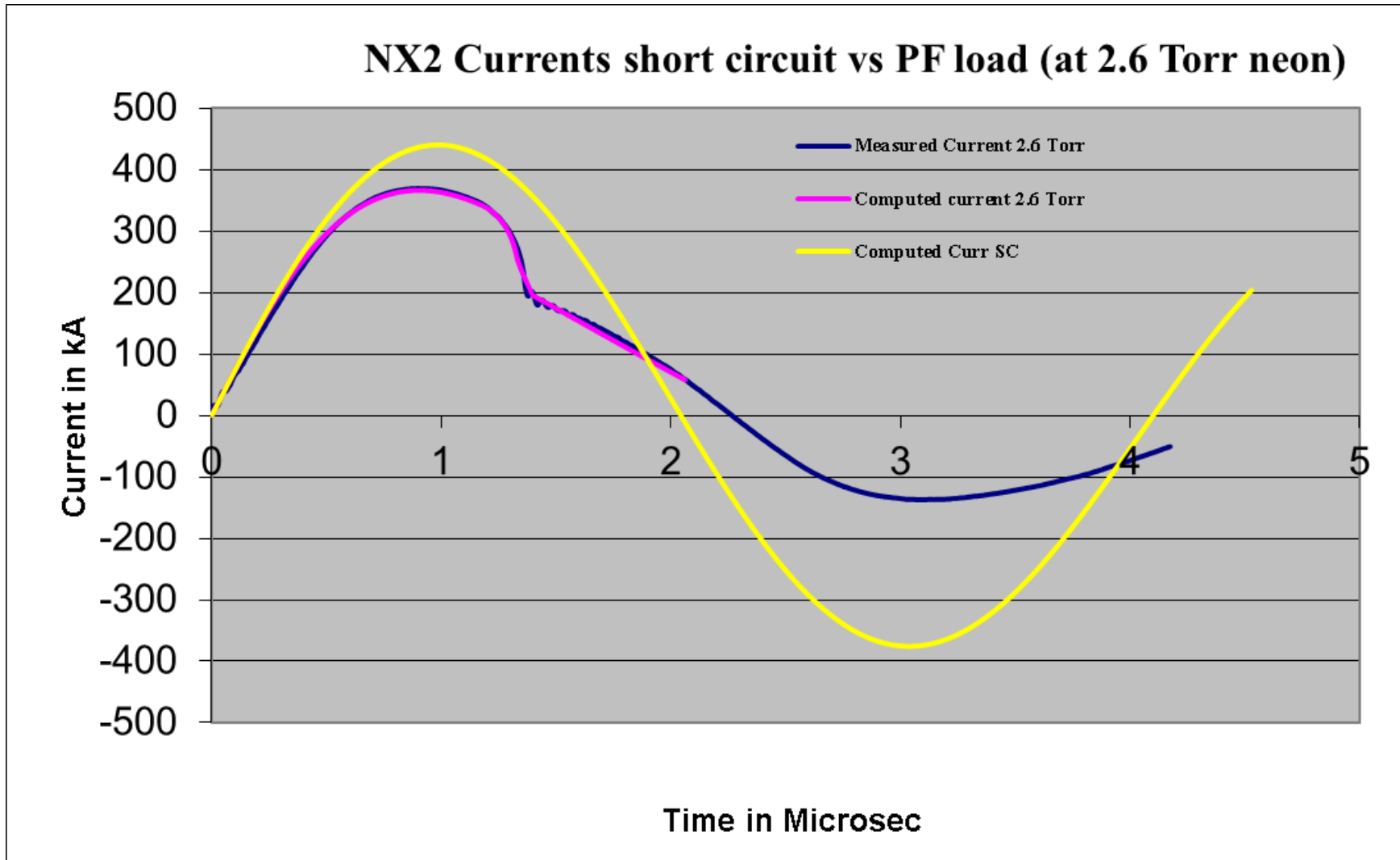
**axial transit time**

$$t_a = \left[ \frac{4\pi^2 (c^2 - 1)}{\mu \ln c} \right]^{1/2} \frac{\sqrt{f_m}}{f_c} \frac{z_o}{(I_o / a) / \sqrt{\rho}} :$$

radial transit time

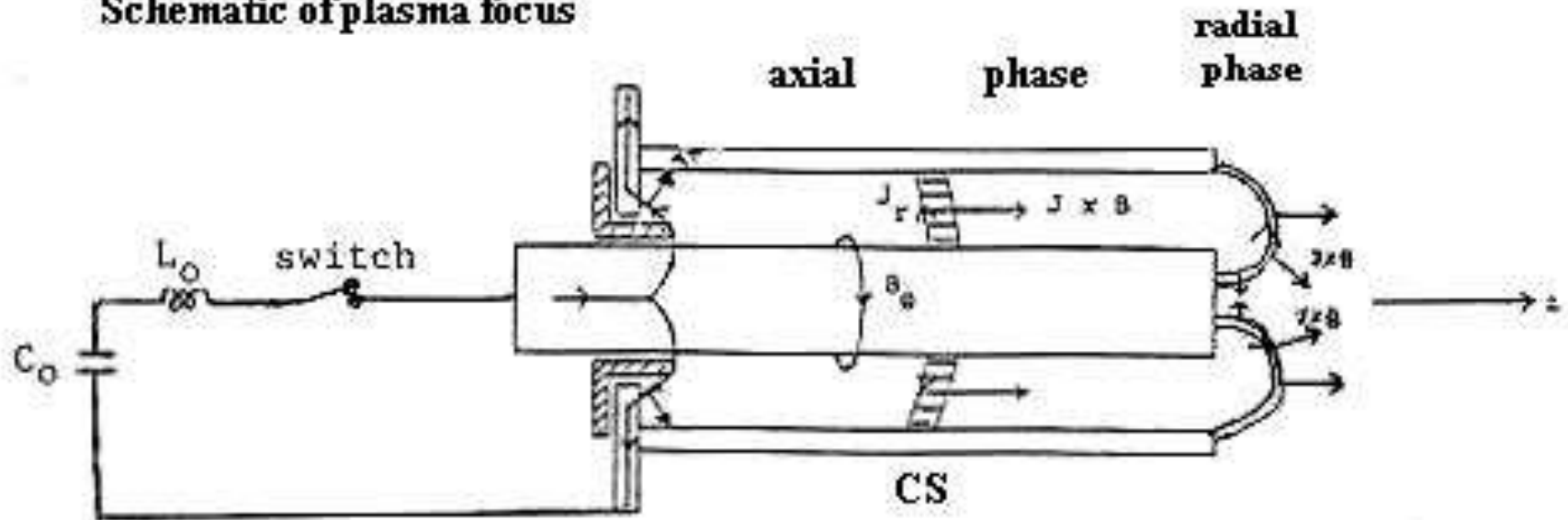
$$t_r = \frac{4\pi}{[\mu (\gamma + 1)]^{1/2}} \frac{\sqrt{f_{mr}}}{f_c} \frac{a}{(I_o / a) / \sqrt{\rho}}$$

# Capacitor bank- no load (SC) vs into PF tube



# Diagnostics from modelling: The Model Schematic

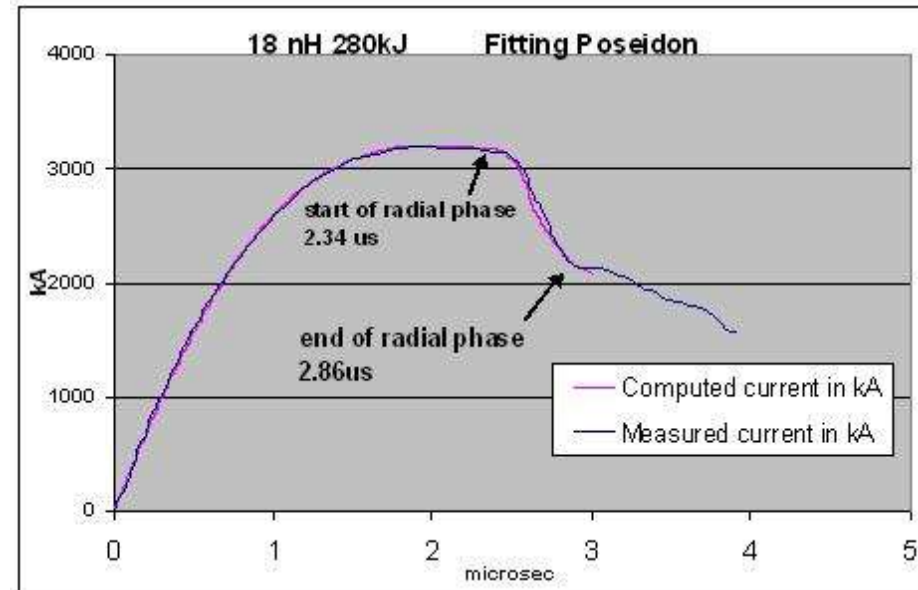
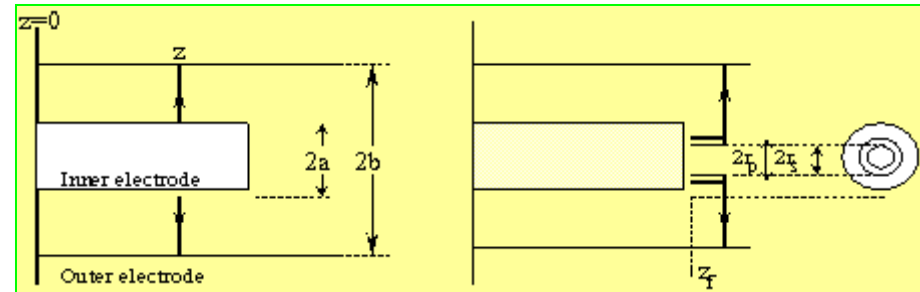
Schematic of plasma focus



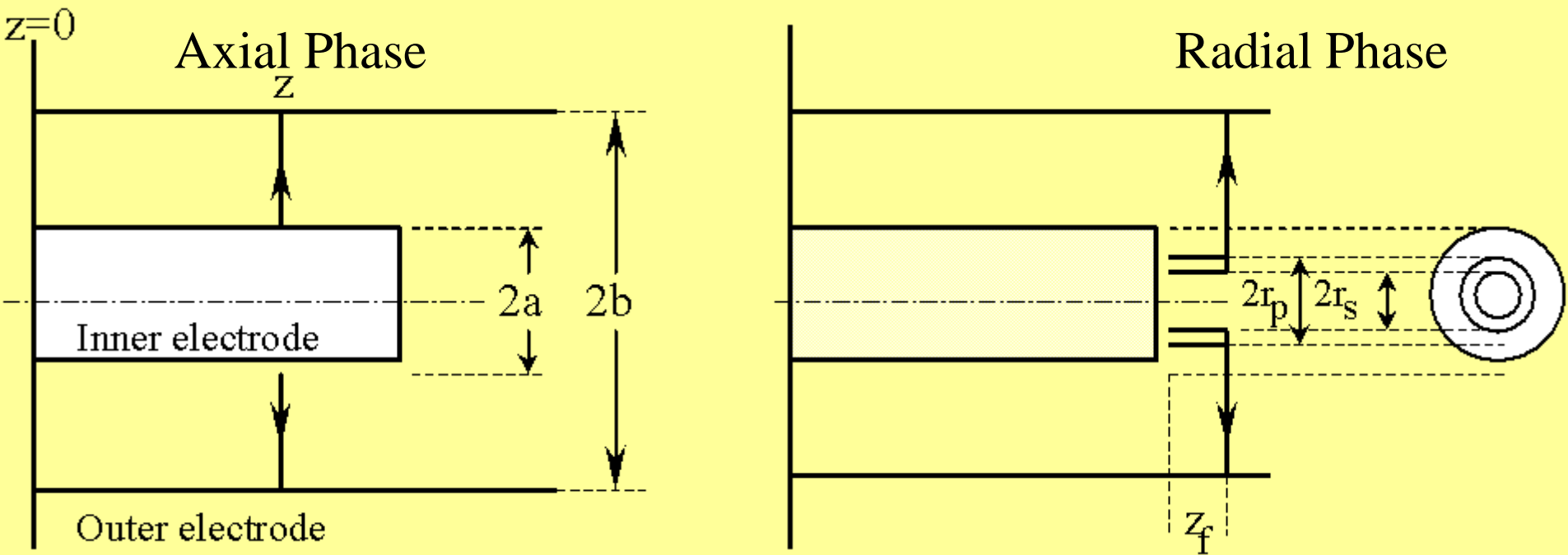
A capacitor bank discharges a large current into the coaxial tube. The current flows in a current sheath CS which is driven by the  $J \times B$  force axially down the tube. At the end of the axial phase the CS implodes axially, forming an elongating pinch.

# THE PLASMA FOCUS

- The PF is divided into two sections.
- **Pre-pinch (axial) section:** Delays the pinch until the capacitor discharge approaches maximum current.
- The pinch starts & occurs at top of the current pulse.
- Equivalent to driving the pinch with a super-fast rising current; without necessitating fast line technology.
- The intensity which is achieved is superior to even the super fast pinch.



# Two Main Phases of the Plasma Focus



# Schematic of Code

Computes:

- **Axial phase-snowplow model:** 2 coupled equations-1 motion, coupled with 1 circuit. Incorporates mass swept-up fraction  $f_m$ , and plasma current fraction  $f_c$ . These model parameters account for all axial effects not specifically modelled.
- **Radial inward shock phase-, Shock Front-Current Sheet slug with thermodynamics:** 4 coupled equations, 3 motion, coupled with 1 circuit; radial mass swept-up factor  $f_{mr}$  and current factor  $f_{cr}$ . These two model parameters are applied for the radial phases
- **Reflected Shock RS Phase**
- **Pinch radiative phase,** including plasma self-absorption & possibility of radiative collapse
- **Post-compression large radius axial phase**

# Axial Phase (I) snow-plow model- **2 coupled equations for 2 variables z and I**

- Motion**

$$\frac{d^2 z}{dt^2} = \left[ \frac{f_c^2}{f_m} \frac{\mu(\ln c)}{4\pi^2 \rho_o (c^2 - 1)} \left(\frac{I}{a}\right)^2 - \left(\frac{dz}{dt}\right)^2 \right] / z$$

- Circuit**

$$\frac{dI}{dt} = \left[ V_o - \frac{\int I dt}{C_o} - r_o I - I f_c \frac{\mu}{2\pi} (\ln c) \frac{dz}{dt} \right] / \left[ L_o + \frac{f_c \mu}{2\pi} (\ln c) z \right]$$

# Additional Equations

- **Temperature** and **density** in axial phase are calculated using shock-jump equations
- **Tube voltage** is calculated

# Analyse these 2 coupled Equations by dimensional analysis

- Replace variables  $t$ ,  $z$ ,  $I$  by non-dimensionalised quantities as follows:
- $\tau=t/t_0$ ,  $\zeta=z/z_0$  and  $\iota=I/I_0$  where  
 $t_0=(L_0/C_0)^{0.5}$  and  $I_0 = V_0/Z_0$  where  $Z_0=(L_0/C_0)^{0.5}$   
is the surge impedance.

## 2 Non-dimensionalised axial phase equations

- Motion

$$\frac{d^2 \zeta}{d\tau^2} = \left[ \frac{f_c^2}{f_m} \frac{\mu \ln c}{4\pi^2 \rho_0 (c^2 - 1)} \left( \frac{I_0}{a} \right)^2 \frac{t_0^2}{z_0^2} r^2 - \left( \frac{d\zeta}{d\tau} \right)^2 \right] / \zeta$$

- Circuit

$$\frac{d^2 \zeta}{d\tau^2} = \frac{\left[ \alpha^2 r^2 - \left( \frac{d\zeta}{d\tau} \right)^2 \right]}{\zeta}$$

# Axial Transit Time & 2 scaling parameters

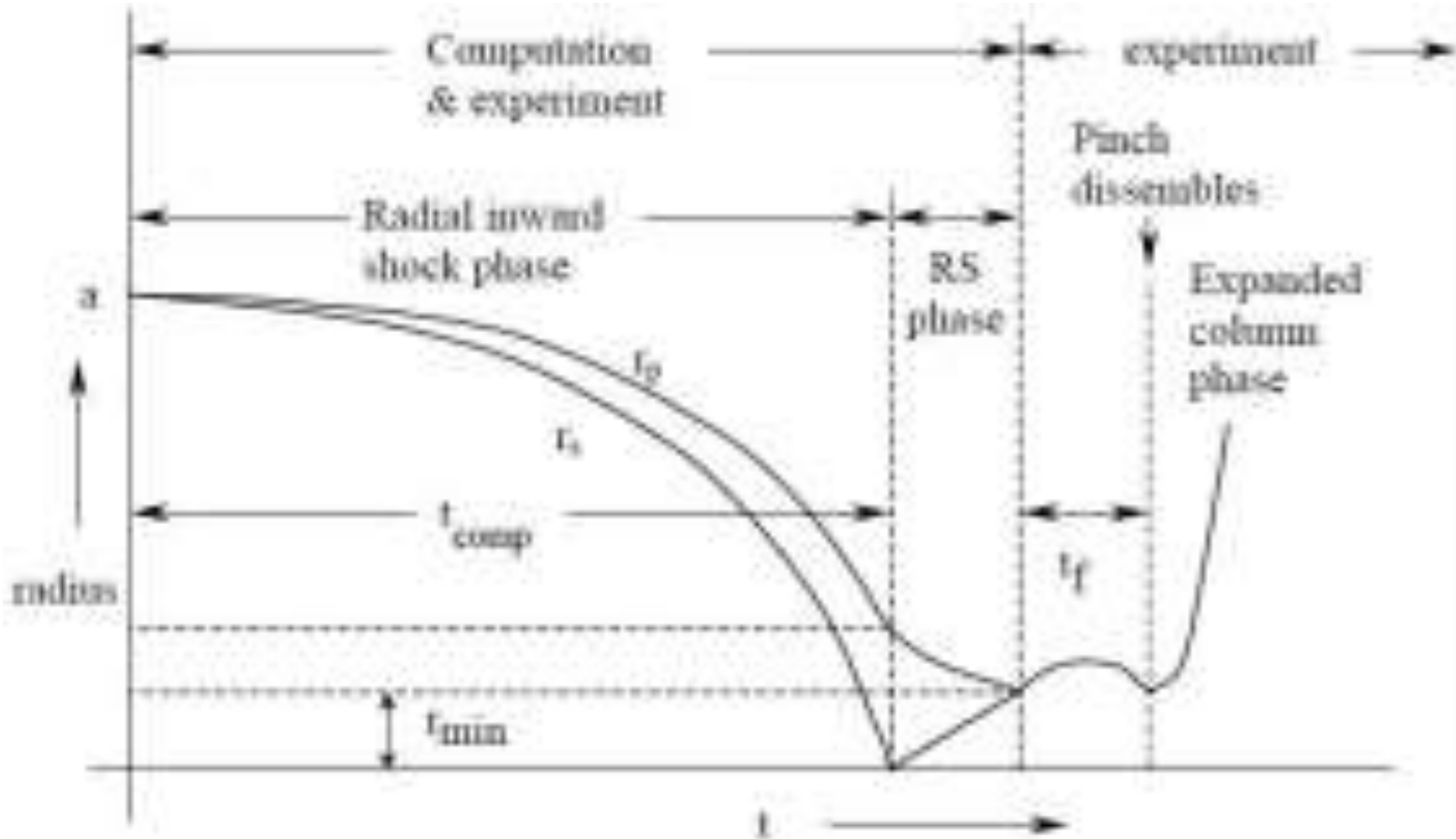
This process identifies the axial transit time as:

$$t_a = \left[ \frac{4\pi^2 (c^2 - 1)}{\mu \ln c} \right]^{\frac{1}{2}} \frac{\sqrt{f_m}}{f_c} \frac{z_o}{(L_o / \alpha) / \sqrt{\rho}}$$

And two scaling parameters:  $\alpha = (t_0/t_a)$  and

$$\beta = L_0/L_a$$

# Radial Phase Slug Model



# Radial phase (II) – 4 coupled equations

4 variables  $r_s$ ,  $r_p$ ,  $z_f$  and  $I$

- radial inward shock speed

$$\frac{dr_s}{dt} = - \left[ \frac{\mu(\gamma+1)}{\rho_o} \right]^{\frac{1}{2}} \frac{f_c}{\sqrt{f_{mr}}} \frac{I}{4\pi r_p^2}$$

- Slug elongation speed

$$\frac{dz_f}{dt} = - \left( \frac{2}{\gamma+1} \right) \frac{dr_s}{dt}$$

- Radial Piston speed

$$\frac{dr_p}{dt} = \frac{\frac{2}{\gamma+1} \frac{r_s}{r_p} \frac{dr_s}{dt} - \frac{r_p}{\gamma} \left( 1 - \frac{r_s^2}{r_p^2} \right) \frac{dI}{dt} - \frac{r_p}{z_f} \left( 1 - \frac{r_s^2}{r_p^2} \right) \frac{dz_f}{dt}}{\frac{\gamma-1}{\gamma} + \frac{1}{\gamma} \frac{r_s^2}{r_p^2}}$$

- Circuit

$$\frac{dI}{dt} = \frac{V_o - \frac{\int Idt}{C_o} - r_o I - f_c \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) I \frac{dz_f}{dt} + f_c \frac{\mu}{2\pi} \frac{z_f}{r_p} I \frac{dr_p}{dt}}{L_o + f_c \frac{\mu}{2\pi} (\ln c) z_o + f_c \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) z_f}$$

# Additional Equations for Radial Phase

- **Temperature** & **density** are calculated using Shock-jump equations. **Tube voltage** is calculated.
- Thermodynamics properties:- **charge number**, **specific heat ratio** are calculated using Corona Model
- Critical **correction for finite communication speed** (small disturbance speed) – an algorithm is applied to make this all-important correction
- The above are also applied in the Reflected Shock Phase (III)

# Normalisation of radial phase equations

- $\tau = t/t_0$ ,  $\iota = I/I_0$  as in axial phase but with  $\kappa_s = r_s/a$ ,  $\kappa_p = r_p/a$ ,  $\zeta_f = z_f/a$  ie. distances normalized to anode radius, instead of anode length.

# Normalised radial equations- 4 coupled equations for 4 variables

- Radial shock motion  $\frac{d\kappa_s}{d\tau} = -\alpha\alpha_1 / \kappa_p$

- Elongation of slug  $\frac{d\zeta_f}{d\tau} = -\frac{2}{\gamma+1} \frac{d\kappa_s}{d\tau}$

- Radial piston motion

$$\frac{d\kappa_p}{d\tau} = \frac{\frac{2}{\gamma+1} \frac{\kappa_s}{\kappa_p} \frac{d\kappa_s}{d\tau} - \frac{\kappa_p}{\gamma} \left(1 - \frac{\kappa_s^2}{\kappa_p^2}\right) \frac{d\tau}{d\tau} - \frac{1}{\gamma+1} \frac{\kappa_p}{\zeta_f} \left(1 - \frac{\kappa_s^2}{\kappa_p^2}\right) \frac{d\zeta_f}{d\tau}}{(\gamma-1)/\gamma + (1/\gamma)(\kappa_s^2/\kappa_p^2)}$$

- Circuit

$$\frac{d\tau}{d\tau} = \frac{1 - \int d\tau + \beta_1 \left[ \ln(\kappa_p / c) \right] \tau \frac{d\zeta_f}{d\tau} + \beta_1 \frac{\zeta_f \tau}{\kappa_p} \frac{d\kappa_p}{d\tau} - \delta \tau}{\left\{ 1 + \beta - (\beta_1) \left[ \ln(\kappa_p / c) \right] \zeta_f \right\}}$$

# Characteristic radial transit time

- Characteristic radial slug transit time **revealed**

**as:**

$$t_r = \frac{4\pi}{[\mu(\gamma+1)]^{1/2}} \frac{\sqrt{f_{mv}}}{f_c} \frac{a}{(I_0/a)\sqrt{\rho}}$$

- 2 scaling parameters

$$\alpha = t_a/t_r$$

$$\beta_1 = \beta/(Flnc)$$

# Reflected Shock (III) Phase, 4 coupled equations for 4 variables, $r_r$ , $r_p$ , $z_f$ & $I$

- Reflected shock speed  $\frac{dr_r}{dt} = -0.3 \left( \frac{dr_s}{dt} \right)_{on-axis}$

- Piston Speed 
$$\frac{dr_p}{dt} = \frac{-\frac{r_p}{\gamma l} \left( 1 - \frac{r_s^2}{r_p^2} \right) \frac{dl}{dt} - \frac{r_p}{z_f} \left( 1 - \frac{r_s^2}{r_p^2} \right) \frac{dz_f}{dt}}{\frac{\gamma - 1}{\gamma} + \frac{1}{\gamma} \frac{r_s^2}{r_p^2}}$$

- Elongation Speed  $\frac{dz_f}{dt} = - \left( \frac{2}{\gamma + 1} \right) \left( \frac{dr_s}{dt} \right)_{on-axis}$

- Circuit 
$$\frac{dl}{dt} = \frac{V_o - \frac{\int l dt}{C_o} - r_o I - f_c \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) I \frac{dz_f}{dt} + f_c \frac{\mu}{2\pi} \frac{z_f}{r_p} I \frac{dr_p}{dt}}{L_o + f_c \frac{\mu}{2\pi} (\ln c) z_o + f_c \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) z_f}$$

# Additional Equations for RS Phase

- Same as for Radial Shock Phase

# Pinch Phase (IV) - 4 coupled equations

with 4 variables -  $r_p$ ,  $z_f$ ,  $I$  &  $Q$

- Radiation-coupled Piston Speed

$$\frac{dr_p}{dt} = \frac{\frac{-r_p}{\gamma I} \frac{dI}{dt} - \frac{1}{\gamma + 1} \frac{r_p}{z_f} \frac{dz_f}{dt} + \frac{4\pi(\gamma - 1)}{\mu_0 z_f} \frac{r_p}{f_c^2 I^2} \frac{dQ}{dt}}{\gamma}$$

- Pinch elongation- similar to earlier equation
- Circuit – similar to earlier equation
- Energy gain/loss  $Q$  coupled into dynamics

# Additional Equations-Pinch Phase

- **Density**- use mass conservation
- **Temperature**- use Bennett balance
- **Plasma Resistance**- use Spitzer equation

# Additional Equations – Pinch Phase

**Q** has following components:

**Joule heat**- Use Ohm equation

**Bremsstrahlung, recombination and line radiation**- use respective radiation power equations; sum for total radiation power

**Plasma self-absorption**- computed using a schematic developed jointly with N A D Khattak

# Additional Equations- Pinch Phase

- **Characteristic radiation** of a gas- Line radiation is integrated within a temperature window based on ionization levels; e.g. Ne SXR yield
- **Neutron yield**- calculated using thermonuclear model + beam-gas target model (calibrated to globally measured yield curve)
- **Beam ions** fluence, flux etc properties,  
**Fast Plasma Stream**- computed from plasma inductive energy conversion.

# Post-Pinch axial phase (V)- 2 coupled equations with 2 variables

- Similar to Axial Phase (I) with objective to complete the computed current waveform

# Integration of equations- scheme

- $t=0$ , set **initial values** for  $d^2z/dt$ ,  $dI/dt$
- **Increment  $t$** , integrate for  $z$  and  $I$ , calculate new values for  $d^2z/dt$ ,  $dI/dt$  etc
- **Point-by-point** sequentially, each phase below:

**Axial phase**

**Radial inward phase**

**RS phase**

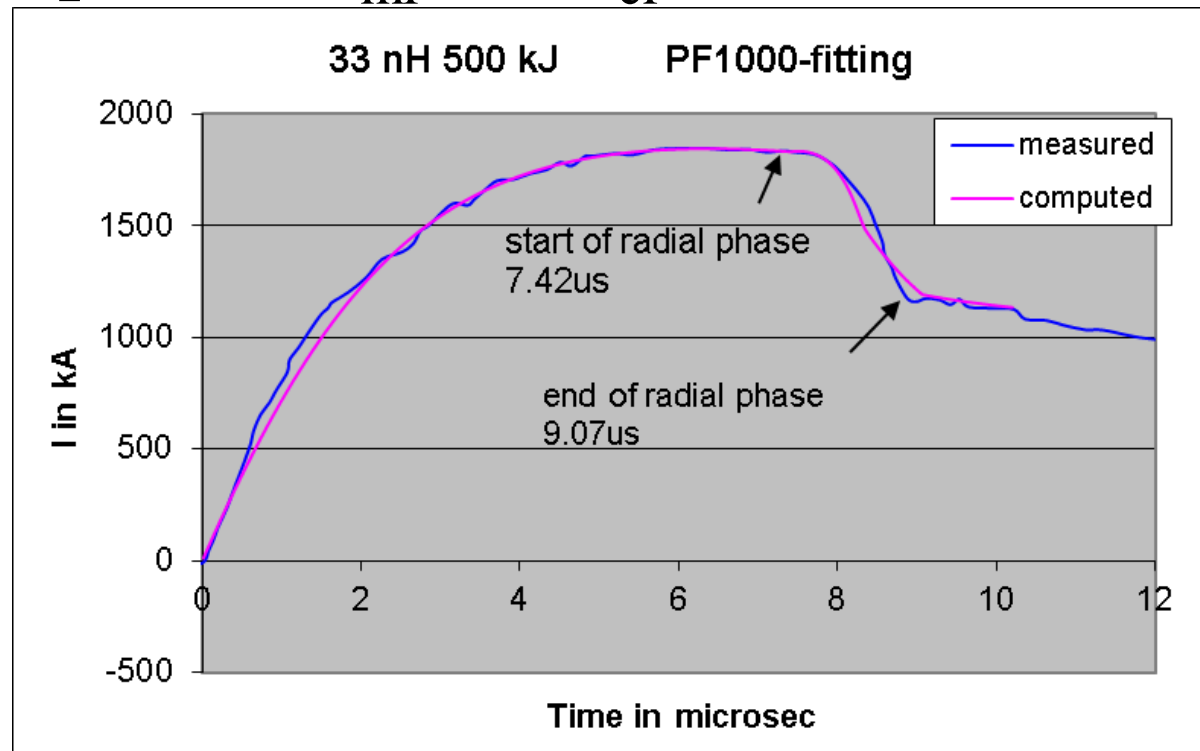
**Pinch phase**

**Post pinch large column axial phase**

# Connecting code and reality- fitting the computed current waveform to the measured waveform

Fitting parameters:

- axial phase:  $f_m$  and  $f_c$
- Radial phases:  $f_{mr}$  and  $f_{cr}$



# Significance of $I_{\text{total}}$ current fitting

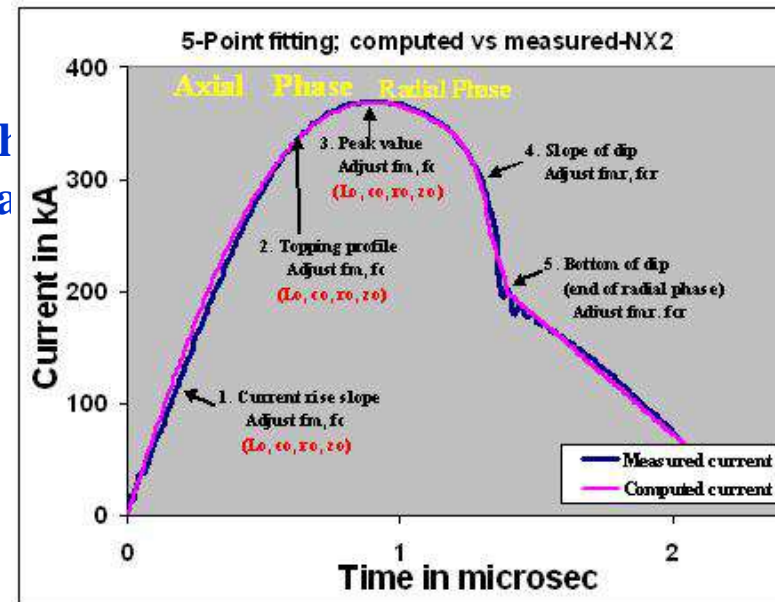
- $f_m$ : mass swept up factor, axial phase  
accounts for all effects affecting mass swept up (structure inclination, porosity, boundary layer etc)
- $f_c$ : plasma current factor, axial phase  
accounts for all effects affecting current flowing in the plasma (current leakage to backwall, shunting and fragmenting, CS inclination etc), defines the fraction of  $I_{\text{total}}$  effectively driving the magnetic piston
- $f_{mr}$ : mass swept up factor, radial phase  
accounts for all effects affecting mass swept up (structure inclination, porosity, axial mass streaming etc)
- $f_{cr}$ : plasma current factor, radial phase  
accounts for all effects affecting current flowing in the plasma (current leakage to backwall, CS bifurcation, current constriction/disruption etc) defines the fraction of  $I_{\text{total}}$  effectively driving the magnetic piston

We fit the computed  $I_{\text{total}}$  waveform to the measured because the  $I_{\text{total}}$  waveform is the one usually measured. Once the  $I_{\text{total}}$  waveform is fitted by adjusting the 4 model parameters, the  $I_{\text{plasma}}$  waveform is also implicitly fitted.

# Fitting Procedure- the 5-point fit

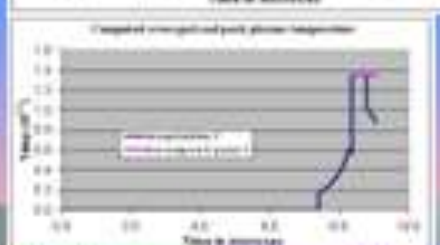
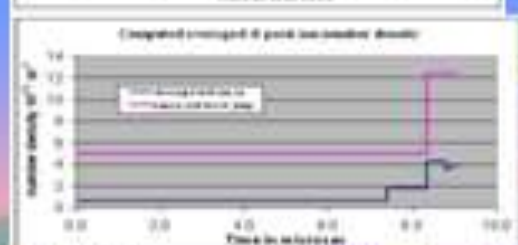
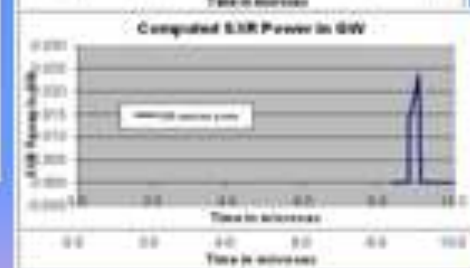
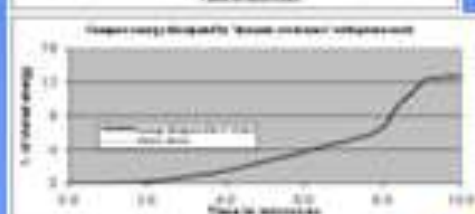
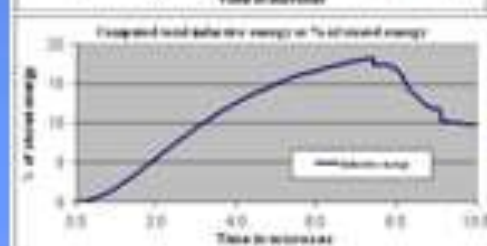
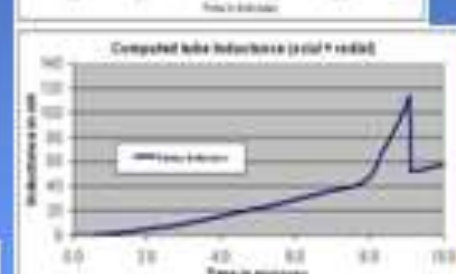
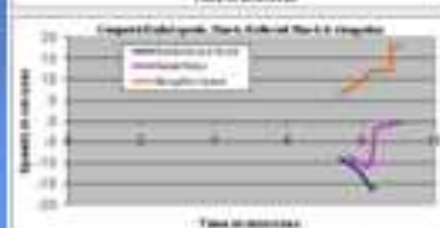
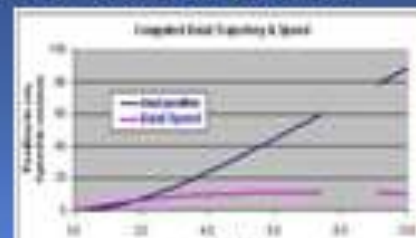
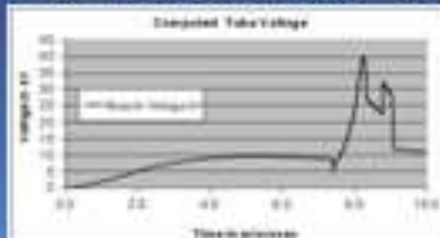
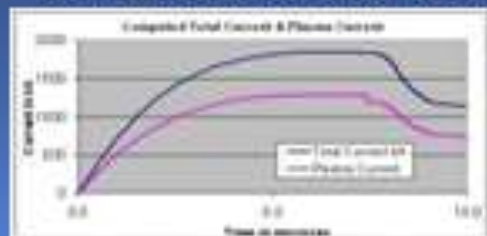
## The 5-Point Fit:

- First, the axial model factors  $f_m, f_c$  are adjusted (fitted) until
  - (1) computed rising slope of the  $I_{total}$  trace and
  - (2) the rounding off of the peak current as well as
  - (3) the peak current itselfare in reasonable (typically very good) fit with the measured current.
- Next, adjust (fit) the radial phase model factors  $f_{mr}$  and  $f_{cr}$  until
  - (4) the computed slope and
  - (5) the depth of the dipagree with the measured  $I_{total}$  waveform.



# Once fitted, the **code outputs the properties** of the PF

Computed Properties of the PF1000: Currents, tube voltage, trajectories, speeds, energy distributions, temperatures, densities, SXR power and neutron yield





# Applications of the Code

- **Reference for diagnostics:** dynamics, pinch size and lifetimes, SXR yields, neutron yields, ion beams and FPS; anomalous resistance, amplitudes and time scales-applied to many PF's, including tapered PF, curved anodes and hemispherical anode
- **Design & optimization:** UNU/ICTP PFF (network) , NX2, NX3, FMPF (Singapore), FNII (Mexico), UBA (Argentina), KSU (USA), FOFU (USA), other machines Chile, Iran, Syria etc, DuPF
- **Special designs:** current-stepped PF, multi-MJ PF
- **ML: modification** of code schematic to Filippov machines

# Insights from code

- **Current and neutron limitations** as inductance is reduced towards small values e.g. 10 nH
- **Scaling properties of PF**: speed, T, density
- Scaling properties for SXR , neutron, beam ions & FPS
- **Scaling laws** of SXR , neutron, beam ions (FIB) & Fast Plasma Streams (FPS)
- **Deterioration** of neutron and radiation yield scaling
- **SFM**- advantageous for materials deposition
- **Radiative cooling and collapse** for HED

# Radiative Cooling and Collapse

- Pease-Braginskii proposed the mechanism of radiative collapse for a Z-pinch in hydrogen - PS current is 1.4 MA
- Our Radiation-coupled dynamic equation:

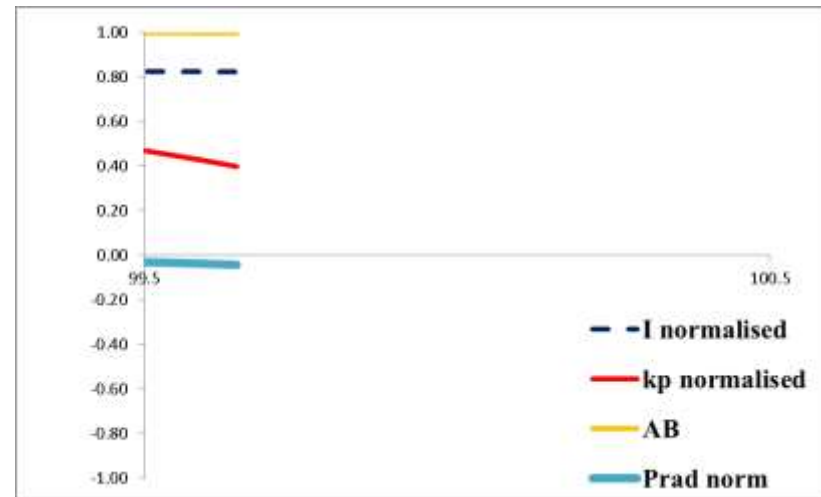
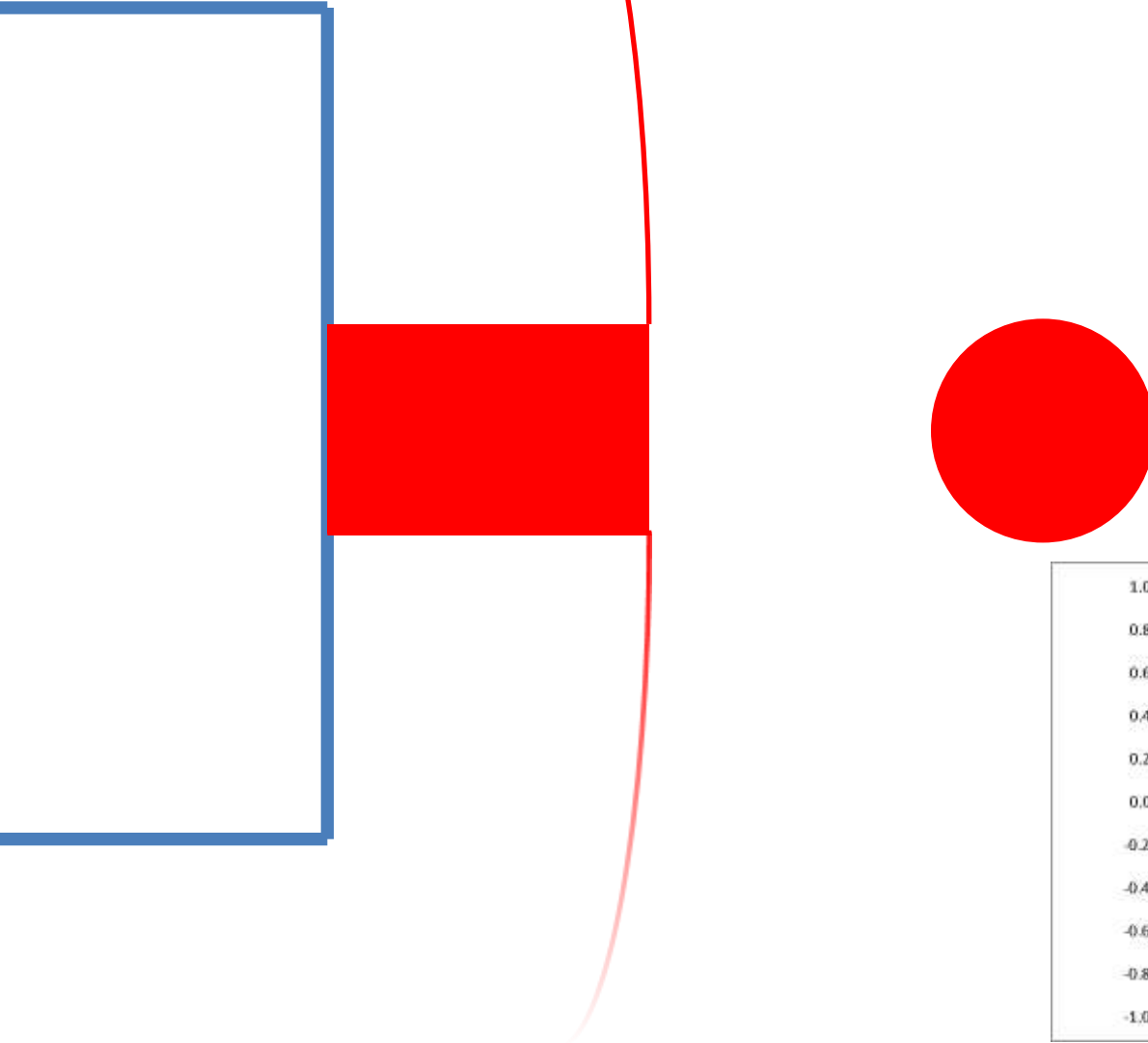
$$\frac{dr_p}{dt} = \frac{\frac{-r_p}{\gamma l} \frac{dl}{dt} - \frac{1}{\gamma + 1} \frac{r_p}{z_f} \frac{dz_f}{dt} + \frac{4\pi(\gamma - 1)}{\mu\gamma z_f} \frac{r_p}{f_c^2 I^2} \frac{dQ}{dt}}{\gamma}$$

If we put the term  $dQ/dt=0$ , and consider the radiation part of  $Q$  to consist only of bremsstrahlung, then we have the P-B limit.

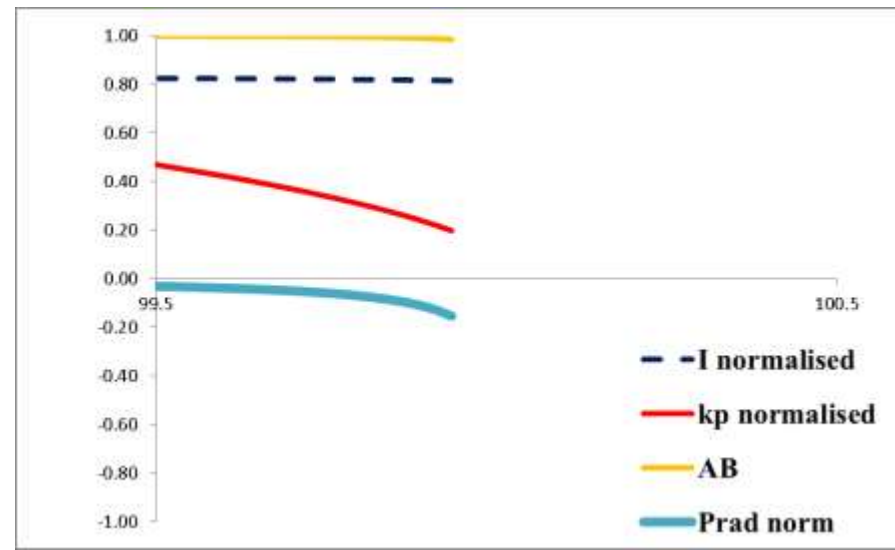
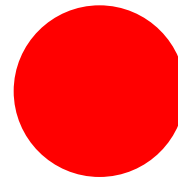
# Reduced P-B limit for high Z gases

- For higher Z gases such as neon, line radiation predominates and since line radiation is many times more in power than bremsstrahlung at PF conditions, the P-B limit current is reduced to less than 100 kA in the case of argon, Kr and Xe.
- The code demonstrates radiative collapse in small devices, including the INTI PF.

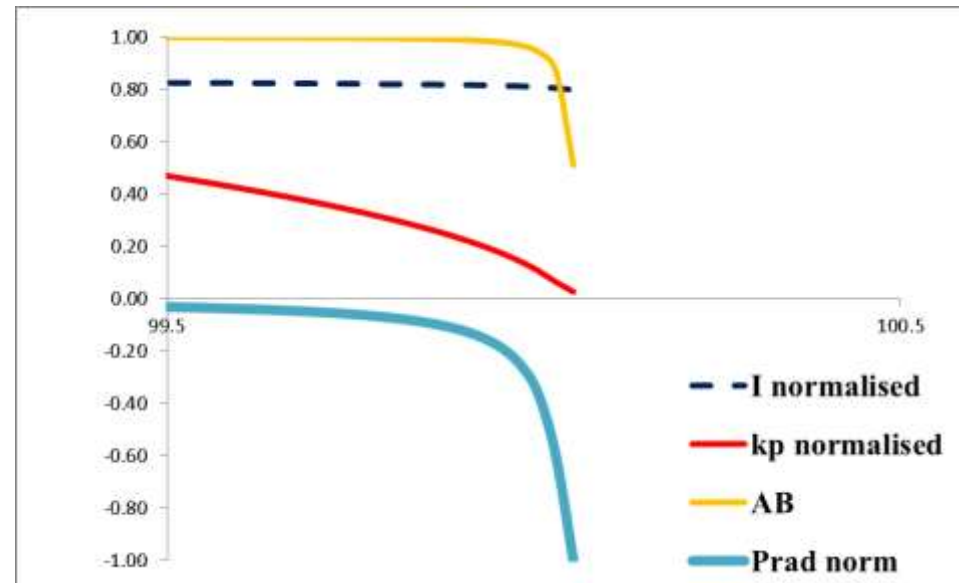
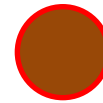
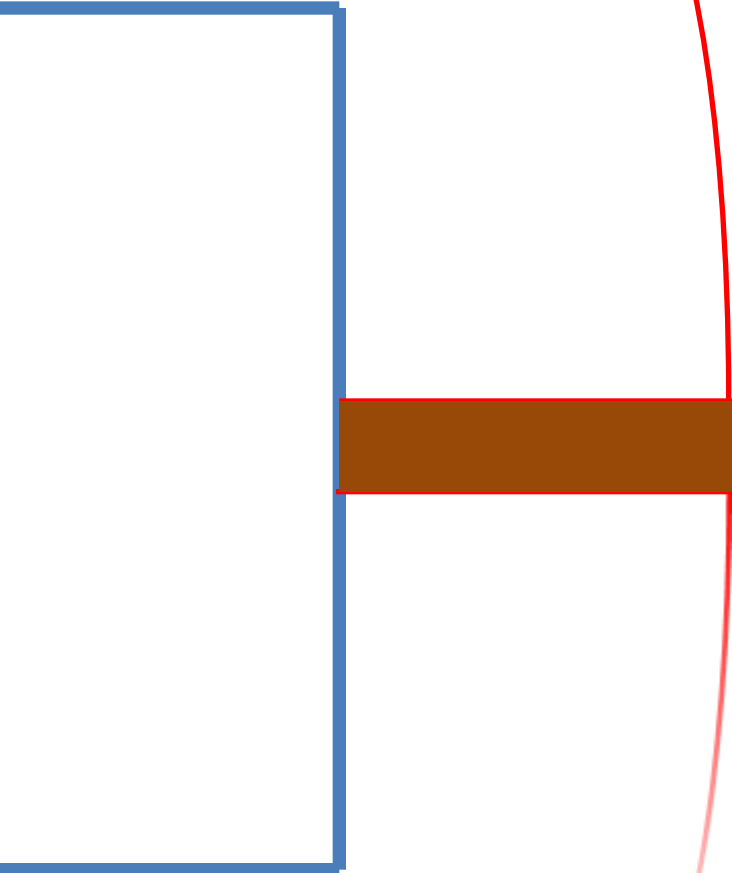
# Bennett Pinch-Plasma Pressure balances Magnetic Field



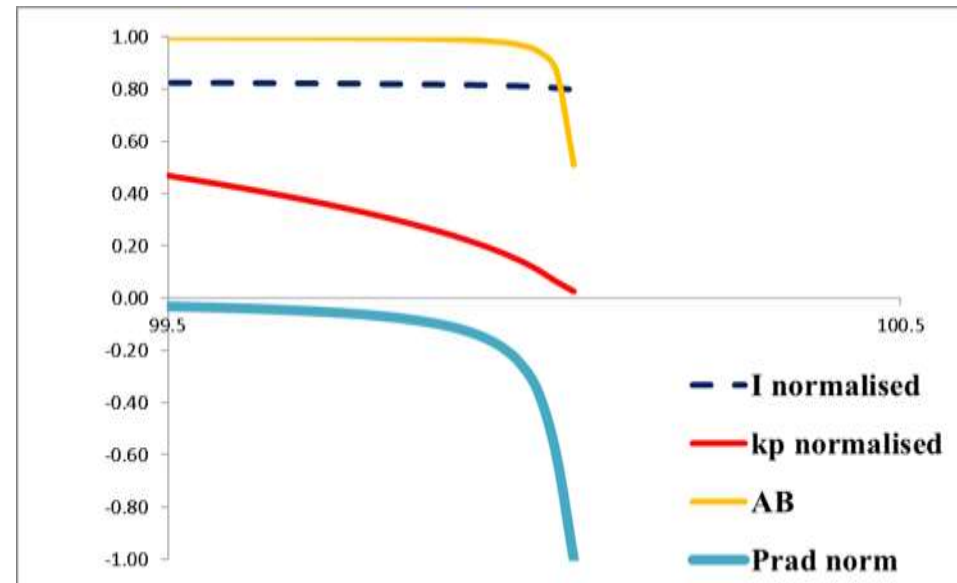
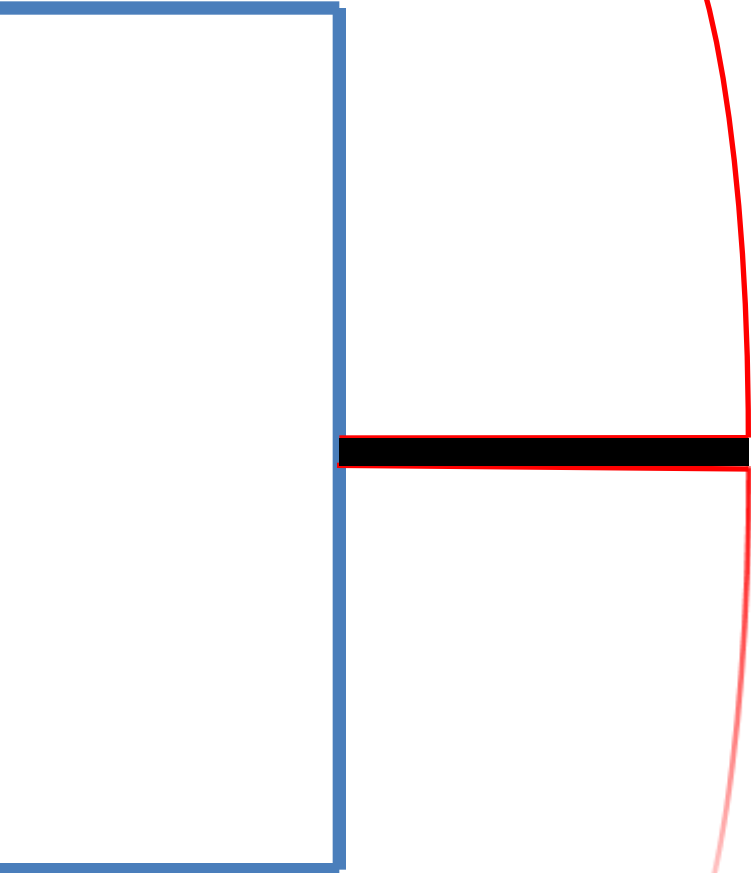
**Radiative collapse starts-** intense radiation cools plasma, leading to lower plasma pressure,  $dQ/dt$  term dominated by radiation



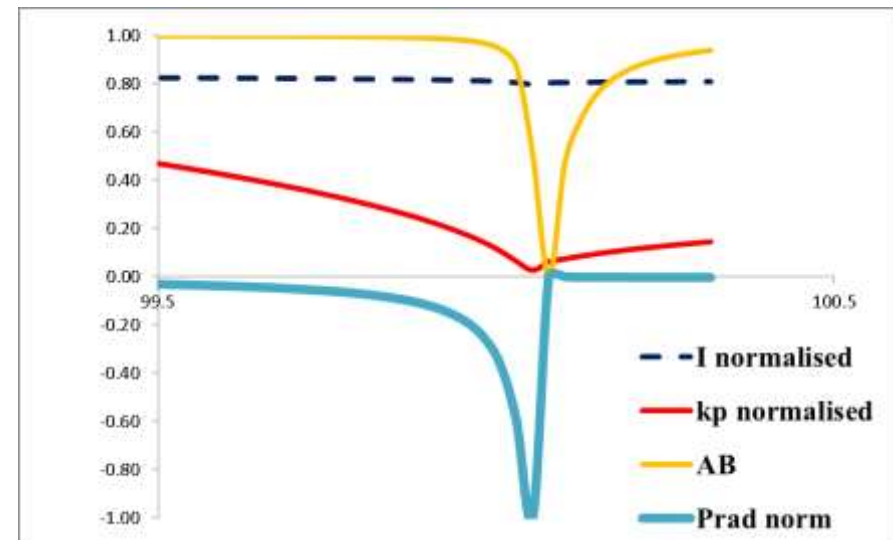
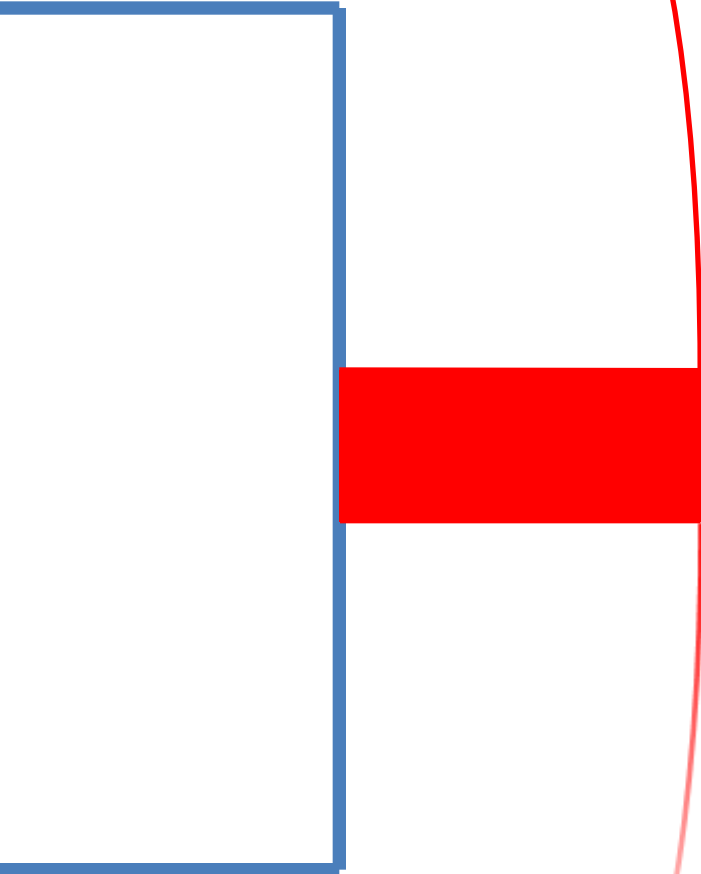
**Radiative collapse continues-** plasma density increases, plasma begins to absorb its own radiation- becoming partially opaque to its own radiation



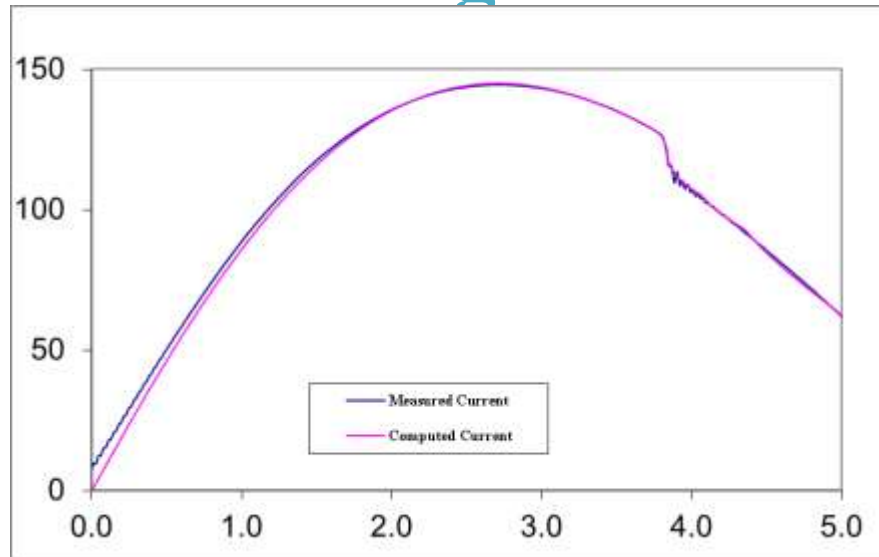
**Radiative collapse stops-** plasma density is so high that plasma absorbs sufficient radiation from its volume to stop the radiative collapse



**Pinch expands-** radiation becomes insignificant;  
Joule heating now dominates  $dQ/dt$  term

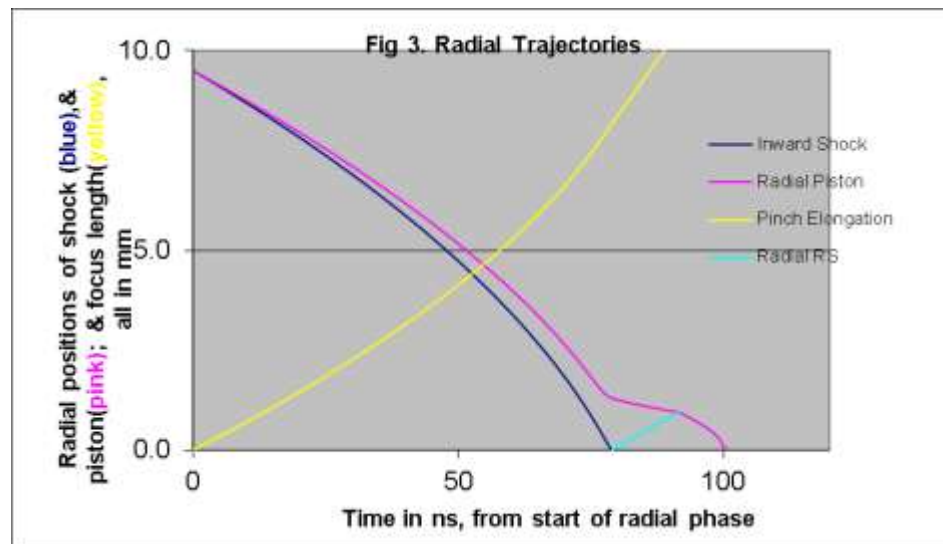


# Observation of radiative collapse in INTI PFCurrent fitting for 0.5 Torr Kr



**Fig 1. Fitting the computed current trace to the measured current trace of INTI PF at 12 kV 0.5 Torr Kr (shot 631).**

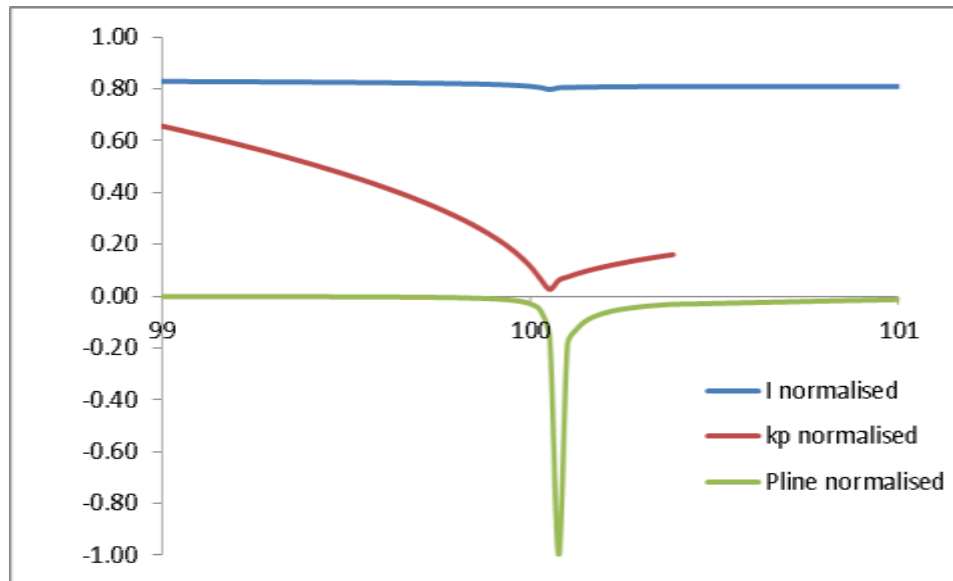
# Computed Radial Trajectory showing Radiative Collapse



**Fig 2 shows the radial trajectory corresponding to the fitting of the current waveform of Fig 1 for INTI PF 12 kV, 0.5 Torr Kr.**

# Correlating Radiative Collapse with Line Radiation Power

Normalisation:  $I$ : 145 kA,  $k_p$ : 0.0014,  $P_{\text{line}}$ : 3.7 TW

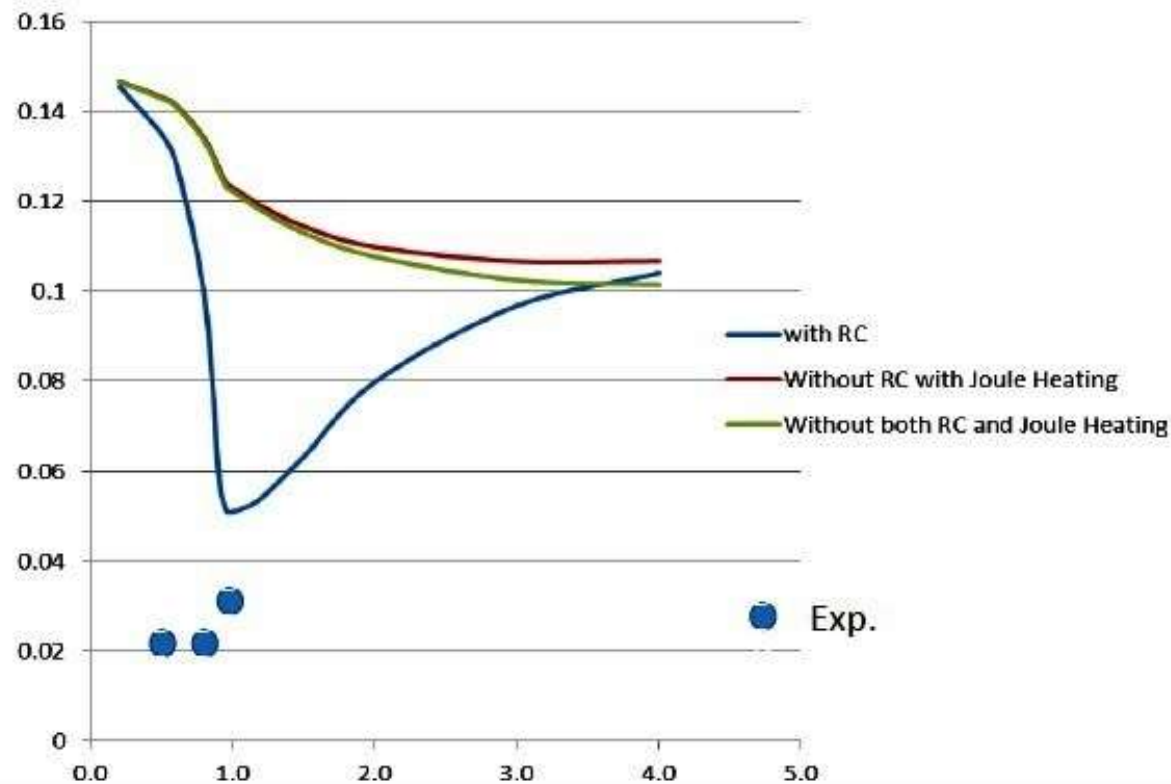


# Correlating RC Trajectory with $P_{\text{line}}$

- The pinch compresses to a radius of 0.0013 cm;  $k_p = 0.0014$ .
- The radiative collapse is ended when plasma self-absorption attenuates the intense line radiation.
- The rebound of the pinch radius is also evident.
- This intense compression, reaches  $3.7 \times 10^{26}$  ions  $\text{m}^{-3}$ , which is 15 times atmospheric density (starting from less than 1/1000 of an atmospheric pressure).
- Energy pumped into the pinch is 250 J whilst 41 J are radiated away in several ns,
- Most of the radiation occurs in a tremendous burst of 50 ps
- Peak radiation power:- almost  $4 \times 10^{12}$  W.
- The energy density at peak compression is  $4 \times 10^{13}$  J  $\text{m}^{-3}$  or 40 kJ  $\text{mm}^{-3}$ .
- Thus even in this small plasma focus intense HED is achieved with immense radiation power.

# PF1000- 23 kV 1.1 MA pinch current: Pinch radius ratio versus pressure: Comparison of code with time-resolved imaging-

\* from Experiments and Simulations on the Possibility of Radiative Collapse in the Plasma Focus  
PF-1000- M. Akel, J. Cikhardt, P. Kubes, H.-J. Kunze, S. Lee, M. Paduch, and S. H. Saw-  
PLASMA 2015, Warsaw, September 7-11, 2015



# Conclusion

- Model:- phases, equations
- Applications:- dynamics and properties, reference for diagnostics, design & optimisation,
- Insights:- scaling properties & scaling laws for SXR, neutrons, FIB & FPS,
  - deterioration of yield scaling
  - Slow Focus Modes with advantages for materials deposition
  - Radiative cooling and collapse for HED