Conditions for Radiative Cooling and Collapse in the Plasma Focus Illustrated With Numerical Experiments on PF1000

Sing Lee, Sor Heoh Saw, Mohamad Akel, Jalil Ali, Hans-Joachim Kunze, Pavel Kubes, and Marion Paduch

Abstract—Reduced Pease–Braginskii currents are estimated for a linear pinch in a range of gases, namely, D, He, Ne, Ar, Kr, and Xe. A characteristic depletion time is defined as the time it takes for the plasma focus (PF) pinch energy to be radiated away. This quantity is used as an indicator for expectation of radiative collapse. The depletion times in various gases are estimated in units of pinch duration. The values indicate that in D and He, the radiation powers are small, resulting in such long depletion times that no radiative collapse may be expected in the lifetime of the focus pinch. In Ne, low tens of percent are radiated and significant cooling and reduction in radius ratio may be anticipated. In Ar, Kr, and Xe, the depletion time is only a fraction of the estimated pinch duration, so radiative collapse may be expected. Numerical experiments are then carried out with a circuit-coupled code, which incorporates radiation-coupled dynamics with PF pinch elongation and plasma self-absorption. The latter eventually limits the radiated power and stops the radiative collapse. These results show the detailed dynamics and confirm the expectations arising from depletion times discussed above.

Index Terms—Plasma focus (PF), PF modeling, plasma pinch, radiation effects.

I. INTRODUCTION

In a Z-PINCH compressed by large electric currents to high densities and temperatures [1], an equilibrium state may be envisaged when the plasma kinetic pressure rises to balance the compressing magnetic pressure, resulting in the pinch achieving an equilibrium pinch radius. This is the pressure balance basis of the Bennett equation [2]. During the compression, work is done on the column leading to a rise in internal energy. By applying energy balance additionally to pressure balance, the equilibrium radius of the pinch may in principle be computed [3]. This minimum pinch radius was computed to be 0.3 for a deuterium Z-pinch compared with the Imperial College observation [4] of 1/3. For Ar, the energy-balance and pressure-balance method computed [5] the radius ratio as 0.18, compared with the observations of 0.17 at temperatures of $2 \times 10^7$ K for the Imperial College low-pressure high-speed Ar Z-pinch. The radius ratio is somewhat temperature dependent due to the compressibility of the gas dependent on the specific heat ratio (SHR) γ of the plasma. The above is for the situation in which the pinch is assumed to be purely electromagnetic with energy input into the pinch arising only through electromagnetic motion effect. (Note: moreover because of the low density and high temperature, this Ar Z-pinch does not undergo radiative collapse and this example should not be compared and does not contradict with other examples of radiatively collapsed Ar pinches discussed later in this paper.) When Joule heating and radiation emission are considered, these will modify pinch dynamics and pinch configuration. Joule heating will increase internal energy requiring a bigger equilibrium pinch radius, while radiation emission will oppose this trend. The power of emitted radiation may exceed that liberated by Joule heating. In such a situation, the magnetic pressure associated with the electric current continues to exert a radially inward squeezing (pinching) force, but the kinetic (resisting) pressure drops due to the excess radiation power (emitted radiation power minus the Joule power gain). This radiation cooling effect, if sufficient, will lead to a sharp enhancement of compression to a very small radius, which could be far smaller than envisaged in the case of the electromagnetic pinch.

In the case of a hydrogen pinch, the plasma is typically far above the fully ionized temperature and the dominant radiation is free-free bremsstrahlung. The bremsstrahlung power $P_{\text{brem}}$ is proportional to $T^{1/2}$, while plasma resistive heating $P_{\text{joule}}$ is proportional to $T^{-3/2}$. Thus, as pinch current is increased and pinch temperature rises, there comes a point when $P_{\text{brem}}$ exceeds $P_{\text{joule}}$. Pease [6] and Braginskii [7] separately showed that in hydrogen, this point may be defined by a critical pinch current referred to as $I_{P-B}$ of 1.4 MA. In such a pinch at equilibrium when the pinch current is raised above 1.4 MA, radiation collapse may occur.
As the compressed density increases and temperature drops due to emitted radiation, plasma self-absorption [8] sets in limiting the emission of radiation. Radiation collapse will stop. This mechanism will place a lower limit on the radius of the pinch.

For the case of high-Z gases such as Ne, Ar, Kr, and Xe, at the typical temperatures encountered in plasma focus (PF) pinches, the dominant radiation emitted is line radiation, with radiating powers typically several orders of magnitude of $P_{\text{brem}}$. The analysis [9], [10] shows that it is thus easier to achieve radiative collapse in high-Z gases compared with the case of hydrogen.

However, the critical current is only one condition for the occurrence of radiative collapse. Another condition would be the magnitude of the excess radiative power $dQ/dt$ (which we call $Q_{\text{dot}}$, where $Q = \text{total is the energy radiated out of the pinch plasma in which less Joule heat is released in the pinch plasma} acting to reduce the energy in the pinch $E_{\text{pinch}}$. We define a characteristic radiative time as $t_{\text{rad}} \sim E_{\text{pinch}}/Q_{\text{dot}}$, which is the time required for all the pinch energy to be radiated away at the rate $Q_{\text{dot}}$. Robson [8] considered this situation for the case of the hydrogen and helium Z-pinches including the effects of opacity. Robson considered a circuit that provided a constant voltage until the pinch collapsed to its limiting the emission of radiation. Radiation collapse will stop. due to emitted radiation, plasma self-absorption [8] sets in in the form of modified Lee to Filippov-type PF devices [21]. The speed-enhanced PF [16] was demonstrated. Radiative collapse [10], extraction of diagnostic data [33], [41]–[45], PF neutron yield calculations [34], [35], current and neutron yield limitations [36], [37], deterioration of neutron scaling (neutron saturation) [38], [39], current-stepped PF [40], and anomalous resistance data [46]–[48] from current signals have been studied using the code [11], [12] or variants. Radiation and particle yield scaling laws [30], [32], [34], [38], [39], [49]–[54] have been deduced.

Incorporated into the code since 2007, the radiation-coupled equation of motion of the current sheath provides the mechanism of radiative collapse, while plasma self-absorption is computed giving the code the mechanism for limiting the collapse. Using this code, the case of radiation collapse had been demonstrated in Kr [10] and extended to Ar and Xe [55]. The code also computed fast ion beams (FIBs) and fast plasma streams (FPSs) in D in a range of devices [49], [50]. In that process, the minimum pinch radius appeared as a computed quantity. A drastic reduction in this radius demonstrated that radiative cooling also plays a clearly perceptible role in the emission of FIBs and FPSs in PF devices having gases with an atomic number as low as 7.

Lee et al. [10] demonstrated that the radiation-coupled piston equation of the code produced the correct Pease–Braginskii (P-B) current of 1.6 MA for a deuterium PF pinch. They also showed that in high-Z gases, there is a reduction in the P-B current due to two mechanisms, one related to the charge number (the charge factor) and the other due to the predominant line radiation in high-Z gases, which are not fully ionized in the PF pinch. According to these calculations for He, the reduced P-B current ($I_p$-Reduced) is 1.2 MA considering only the charge factor though there may be a further reduction due to line radiation. However, running the code for PF1000 [56] at 40 kV (in principle the maximum operating voltage of PF1000) in He, the pinch current exceeds 1 MA, but there is no sign of radiative collapse. Even by hypothetically increasing the PF1000 operating voltage to 100 kV when the pinch current exceeds 2 MA, there is still no sign of a sharp drop in the pinch radius ratio, which is the most indicative sign of radiative collapse. To explain this, we develop an expression for the characteristic time required to radiate away the pinch energy through bremsstrahlung and also for the characteristic time for line radiation. The numerical experiments show that the pinch duration has to be of the order (typically at least 0.1) of the characteristic time of radiation ($t_{\text{rad}}$) in order for that radiation to cause significant radiative cooling resulting in radial collapse.

II. RADIATION-COUPLED DYNAMICS FOR THE MAGNETIC PISTON DEFINITIONS

The code uses the following equation for the piston position $r_p$ [10]–[12] derived from the first law of thermodynamics:

$$
\frac{dr_p}{dt} = \frac{r_p}{r_1} \frac{dI}{dt} - \frac{1}{(\gamma + 1) z_f} \frac{dP}{dz} + \frac{3\gamma}{\gamma + 1} \frac{r_p^2}{\gamma z_f^2} \frac{dQ}{dt}
$$

(1)

where $I$ is the total discharge current in the circuit, $f_c$ is the fraction of current flowing into the pinch, $z_f$ is the time-varying length of the PF pinch, and $\gamma$ is the SHR of the plasma. When $dQ/dt$ is negative, energy is lost from
the plasma, adding a negative component to $\frac{dr_p}{dt}$, which tends to reduce the radius $r_p$.

### A. Reduced Pease–Braginskii Current

Following [10], we write the reduced P-B current $I_{p-\text{Reduced}}$ as:

$$I_{p-\text{Reduced}} = I_{p-B}^2 - \frac{1}{K} \times Z'$$  \hspace{1cm} (2)

where $Z' = \left(\frac{1}{4}\right)(1 + Z_{\text{eff}})$ and

$$K = \left[\frac{(dQ_{\text{line}}/dt) + (dQ_{brem}/dt)}{(dQ_{brem}/dt)}\right].$$  \hspace{1cm} (3)

We consider the following powers (all quantities are in SI units unless otherwise stated) of Joule heating, bremsstrahlung radiation, and line radiation, respectively, generated in a plasma column of radius $r_p$ and length $z_p$ at temperature $T$:

$$P_J = C_J T^{-3/2} \frac{Z_p}{r_p} Z_{\text{eff}} I^2_p$$  \hspace{1cm} (4)

$$P_{brem} = C_1 T^{1/2} n_i^2 Z_{\text{eff}}^2 \pi r_p^2 z_p$$  \hspace{1cm} (5)

$$P_{\text{line}} = C_2 T^{-1} n_i^2 Z_p^2 Z_{\text{eff}} \pi r_p^2 z_p$$  \hspace{1cm} (6)

where $C_J \approx 1300$, $C_1 = 1.6 \times 10^{-40}$, and $C_2 = 4.6 \times 10^{-31}$.

For He, the factor $Z' = 0.56$. This factor alone reduces the P-B current to 1.2 MA, even if we assume that He is completely ionized with insignificant line radiation so that $K = 1$. When line radiation becomes dominant, the calculation of $K$ is complicated by the dependence of $P_{\text{line}}$ on density and temperature so that there is no one value for $I_{p-\text{Reduced}}$.

We take some likely points of operation for the gases Ne, Ar, Kr, and Xe and estimate typical values of $I_{p-\text{Reduced}}$ for these gases in Table I. In the example for Ne, we take a typical point of operation for intense line radiation at $Z_{\text{eff}} \sim 9$ so that $Z' \sim 0.31$. At this point of operation, $P_{\text{line}}$ is found to be $20 P_{brem}$ so that $I_{p-\text{Reduced}} \sim 190$ MA. It is emphasized that unlike the value for H or D, which is derived by balancing $P_J$ and $P_{brem}$ resulting in a value dependent only on the pinch current, when higher-Z gases are considered with line radiation that needs to be included the factor $K$, then there is no one value for $I_{p-\text{Reduced}}$. Indeed if we consider a pinch of high-Z gas at high enough temperature that the high-Z plasma is becoming fully ionized, then the value of $I_{p-\text{Reduced}}$ for that pinch has moved up toward the unreduced P-B value. Table I thus gives only indicative values of $I_{p-\text{Reduced}}$ with the trend that as the $Z$-number increases, a lower value of $I_{p-\text{Reduced}}$ may be expected. In particular, He may have a smaller $I_{p-\text{Reduced}}$ than indicated in Table I, which for simplicity has considered only bremsstrahlung for He.

We note that in deriving Table I, the radiation powers are considered at source. The derived $I_{p-\text{Reduced}}$ is indicative of the situation when the plasma is assumed to be completely transparent to the radiation.

#### B. Characteristic Times of Radiation

We also write down the thermal energy in the pinch as the total number of particles in the pinch multiplied by the thermal energy per particle

$$E_{\text{pinch}} = [kT/(\gamma - 1)]n_1(1 + Z_{\text{eff}})\pi r_p^2 z_p$$  \hspace{1cm} (7)

where $\gamma$ is the SHR, which may be written in terms of the degree of freedom $f$ as $\gamma = (2 + f)/f$ so that $1/(\gamma - 1) = f/2$.

In (7), the energy of the pinch is written in a form suitable for high-Z gases in which the energy expended in ionization is not insignificant compared with the translational modes even at the high temperatures concerned. Note that for a fully ionized gas at such a high temperature that the expended ionization energies are already insignificant compared with the translation energy, then $f = 3$ and $[kT/(\gamma - 1)] = 3(kT/2)$ per particle, $k = 1.38 \times 10^{-23}$ being the Boltzmann constant (all quantities are in SI units unless otherwise specified). For example, for gases such as Ne in the PF pinch, the temperature may typically be high enough for it to be approaching full ionization; the SHR computes [5] to be 1.5 so that $f = 4$ and $[kT/(\gamma - 1)] = 4(kT/2)$ per particle. In Kr operating at a temperature of $10^6$ K, $Z_{\text{eff}} \sim 14$, $\gamma \sim 1.3$, $f \sim 6.7$, and $[kT/(\gamma - 1)] = 6.7(kT/2)$ per particle.

We divide the pinch energy by the radiation power to give us a measure of the characteristic time it would take the pinch to have its energy radiated away by that radiation power taken as constant over the whole duration. We call this the characteristic depletion time of radiation.

### C. Characteristic Depletion Time for Bremsstrahlung

We derive the characteristic depletion time due to bremsstrahlung $t_{brem}$

$$t_{brem} = E_{\text{pinch}}/P_{brem} = [kT^{1/2}/(C_1 n_0 f_n)](1 + Z_{\text{eff}})/[Z_{\text{eff}}^3 (\gamma - 1)]$$  \hspace{1cm} (8)

$$= (k^{1/2} / C_1) [1/(n_0^3 f_n^2 z_p^2)] (1 + Z_{\text{eff}})^{1/2}/[Z_{\text{eff}}^3 (\gamma - 1)].$$

Here, we have eliminated $T$ using the Bennett equation for a pinch in which magnetic pressure balances the kinetic pressure: $T = b(I^2/(n_i r_p^2)(1 + Z_{\text{eff}}))$, where $b = \mu k(8\pi^2 n)$ and $\mu = 4\pi \times 10^{-7}$ is permeability so that $b = 1.15 \times 10^{15}$.

The pinch number density $n_i$ is written in terms of the initial number density $n_0$ by writing $n_i = n_0 f_n$, where $f_n = (a r_p)^2 f_{mf} f_g$, accounting not only for the area of compression $(a r_p)^2$ but also for the mass fraction swept in $f_{mf}$ and a geometrical factor $f_g$ due to the elongation of the radial collapse.
To get an estimate of the size of $t_{\text{brem}}$, we put in typical numbers for operation at a pinch current higher than the P-B current for D into (8) as follows: $I = 2.1 \times 10^6$ operated at 3 torr D so that $n_0 = 10^{23}$, $a = 0.2$, $r_p = 3 \times 10^{-2}$ (i.e., $k_{\text{min}} = 0.15$), $f_m = 0.2$, and $f_g = 1/3$ so that $f_n \sim 3$, $\gamma = 5/3$, and $Z_{\text{eff}} = 1$.

For these parameters, $t_{\text{brem}} \sim 1 \times 10^{-3}$ s. This means that the magnitude of $P_{\text{brem}}$ at constant value is such that it would take $10^{-3}$ s to radiate away the pinch thermal energy. Even to radiate away 10% would take 100 $\mu$s. The lifetime of such a PF pinch (e.g., PF1000) may typically be estimated as 0.2 $\mu$s. Thus, in the lifetime of such a PF pinch, it is unlikely that the radiation would affect the dynamics. Looking at (8), we could possibly increase the effect of bremsstrahlung by increasing the ambient pressure within a range suitable for operation.

Careful examination of a large range of numerical experiments shows no sign of radiative cooling in D in which the radiation would affect the dynamics. Looking at (8), we could possibly increase the effect of bremsstrahlung by increasing the ambient pressure within a range suitable for operation.

D. Characteristic Depletion Time for Line Radiation

We derive the characteristic depletion time for line radiation: $t_{\text{line}} = E_{\text{pinch}}/P_{\text{line}} = (k/C_2)I^2/(n_0f_n)$, $1 + Z_{\text{eff}}/[(Z_{\text{eff}}Z_n^4(\gamma - 1)]$ and eliminating $T$.

$$t_{\text{line}} = (kb^2/C_2)I^4/[n_0^2f_n^2r_p^4(1 + Z_{\text{eff}})Z_n^4(\gamma - 1)].$$

Equation (9) shows how depletion times for $t_{\text{line}}$ for a typical PF operation may be computed.

E. Characteristic Depletion Time $t_Q$

In the same way, the net depletion time $t_Q$ may also be computed, where $t_Q$ is the ratio $E_{\text{pinch}}/Qdot$, where $Qdot$ or $dQ/dt = P_{\text{pinch}} + P_{\text{brem}} - P_J$. This $t_Q$ is the time that is applicable. In Table II, we show the sample computations of depletion times in D, He, Ne, Ar, Kr, and Xe for some conditions shown to be a practicable PF operation in the numerical experiments. We modeled the PF configuration after PF1000. For D and He, we operate at (hypothetical) 90 kV in order to reach pinch current in excess of 2 MA. For the other gases, we operate the numerical experiments at 23 kV, which is a voltage that is currently used in the actual PF1000 operation.

In Table II, we calculated depletion times $t_Q$ and also $t_{Q_{\text{eff}}}$, which is $t_Q$ expressed in units of a characteristic pinch time $\tau_{\text{pinch}}$. We took the pinch time as proportional to the anode radius [57]–[59] with a figure of 10 ns/cm (rounding $\tau_{\text{pinch}}$ to 100 ns) as an average value across all the gases. From Table II, it may be surmised that even though the PF is operated with currents above the reduced P-B, and nevertheless, there would be no radiative collapse to be expected from the operation in D and He. In Ne with a significant proportion of pinch energy radiated away within one $\tau_{\text{pinch}}$, radiative cooling should be expected, leading to a considerable reduction in the minimum radius ratio. In Ar, Kr, and Xe, one would expect strong radiative collapse. It is stressed that these numbers act only as a guide since the pinch system is nonstatic and the various properties are interacting continuously. Moreover, all the above estimates are based on the radiative terms at source without the consideration of plasma opacity, which in those cases when the plasma is not completely transparent would reduce the energy loss from the plasma.

We next carry out numerical experiments with the code in which $Q$ and $Qdot$ and plasma self-absorption effect are all included [60] with a smoothed transition from opacity-corrected volume emission to surface emission when opacity effects exceed a set limit. The code models all these effects and properties in a properly coupled interactive fashion.

III. NUMERICAL EXPERIMENTS ON PF1000: RESULTS AND DISCUSSION

A. Fitting for Model Parameters

We have a recent measured current waveform for PF1000 operated at 23 kV at 1.5 torr deuterium. In order to obtain the model parameters, we use the following configuration [11], [12] for PF1000.

1) The bank parameters are $L_0 = 33$ nH (fitted), $C_0 = 1332 \mu$F, and $r_0 = 3$ m$\Omega$ (fitted); tube parameters are $b = 16$ cm, $a = 11.55$ cm, and $z_0 = 60$ cm; the operating parameters are $V_0 = 23$ kV and $P_0 = 1.5$ torr; and the gas parameters (for deuterium) are 4 (molecular weight), 1 (atomic number), and 2 (molecular gas).

We achieved a reasonably good fit (see Fig. 1) by confirming the above bank and tube parameters and obtaining the following model parameters [61]–[63]: $f_m = 0.11$, $f_c = 0.7$, $f_{\text{int}} = 0.26$, and $f_{\text{ct}} = 0.68$.

![Fig. 1. Fitting the computed current trace to the measured current trace of PF1000 at 23 kV, 1.5 torr deuterium. (Note that the two curves have a close fit except after the bottom of the current dip. Fitting is done only up to the bottom of the dip, so any agreement or divergence of the computed and measured traces after the bottom of the dip has no significance.)](image-url)
We then used these model parameters and the above-mentioned configuration for a series of numerical experiments. For all the gases, we operated the numerical experiments at 23 kV, which is a voltage that is currently used in the actual PF1000 operation.

B. PF1000 With D: 23 kV, 3 torr

Fig. 2(a) shows the total discharge current rising to a peak value of 1836 kA. The pinch current $I_{\text{pinch}}$ at the start of pinch (time of start of pinch) is shown with the right-pointing arrow in Fig. 2(a). Fig. 2(b) shows the trajectories in the radial phase. The piston trajectory delineates the pinch radius after the piston meets the reflected shock (RS). For this shot, the pinch lasts 206 ns. The code computes the radial trajectory up to this point. Fig. 2(b) shows a very slow compression (i.e., radius decreases barely perceptibly), typical of an efficiently operated pinch with no radiation compression or significant cooling. A careful study of the computed properties agrees with Table II (and Table III) showing that the radiation power is too small to affect the trajectory. The minimum radius is 22.2 mm. In an extension to this exercise, we have increased the charging voltage in this experiment to a hypothetical PF1000-like configuration of 90 kV with $a = 20$ cm, $b = 28$ cm, and $P_0 = 3.5$ torr. The pinch current is 2.1 MA. This experiment and other numerical experiments carried out earlier [38], [39] confirmed that despite the P-B current being far exceeded, there is no sign of radiative collapse in the D PF.

C. PF1000 With He: 23 kV, 3 torr

The results for He [Fig. 3(a) and (b)] are very similar to the case of D. The minimum radius is 20.5 mm, which is a little smaller than that achieved in the very similar D discharge. This is in agreement with Table II (and Table III) showing that the radiation power in He is not sufficient to severely affect the pinch compression but is larger than that of D and perhaps enough to reduce the minimum radius slightly from that of the case of D. We also carried out extension experiments for He to 90 kV to push the pinch currents above 2 MA. No sign of radiative collapse was found and the radial dynamics in the pinch phase show only the same slow small compression.

D. PF1000 With Ne: 23 kV, 1 torr

In Ne, the effect of radiation on the radial compression of the pinch is unmistakable in both the current waveform and the radial piston trajectory. The total discharge current shows an additional steepening in the final part of the dip (perceptible even without magnifying the relevant region) from a pinch current value of 819 kA at 9.021 $\mu$s [see the right-pointing arrow in Fig. 4(a)] to a value of 673 kA at 9.243 $\mu$s [see the left-pointing arrow in Fig. 4(a)]. A detailed study of the code outputs shows that over this pinch period of 222 ns, the pinch compressed from 15.8 to 7.3 mm [Fig. 4(b)]. These features also correlate with the emission power time profile. Undoubtedly, strong radiative cooling is exhibited in the Ne pinch plasma, leading to a substantial reduction in the pinch radius.

E. PF1000 With Ar: 23 kV, 0.5 torr

Fig. 5(a) and (b) shows the total discharge current $I$ for Ar and the corresponding radial trajectories including the piston trajectory $r_p$. In Fig. 5(c), $I$, $\frac{dQ}{dt}$ (Qdot), and $r_p$ are presented together in normalized scale for the last 600 ns of the radial phase.

At the start of the radial phase, the total circuit current is 1960 kA [Fig. 5(a)], while the plasma current is 1330 kA.
In the first hundreds of nanoseconds of the radial phase, the radial shock front (SF) moves toward the axis driven by the piston with an annular slug of plasma between the SF and the piston [Fig. 5(b)]. The SF hits the axis at 874 ns [see Fig. 5(b) and also the first turn in the \( r_p \) curve of Fig. 5(c)] and because the conditions are collisional, an RS forms and moves radially outward, while the piston continues to move radially toward the axis. The RS hits the piston at 1120 ns [see Fig. 5(b) and also the second turn in the \( r_p \) curve of Fig. 5(c)] at the radial position of 14.8 mm. This is the start of the (normally slow-compression) pinch phase. The pinch current is 820 kA and the net radiative power is \( \frac{dQ}{dt} = 3.0 \times 10^{11} \) W [see Fig. 5(c)]. This is enough to start a rapid compression of the pinch. As the compression proceeds, \( Qdot \) rises to a peak value of \( 7.6 \times 10^{11} \) W at 1143 ns. The plasma self-absorption coefficient \( Ab \) is 0.49 at this time of peak \( Qdot \), and rapidly increasing it starts to reduce the emission of radiation.

The pinch has a radius of 6.2 mm. Even as the power \( Qdot \) decreases [see Fig. 5(c)], the compression continues until a minimum radius of 1.8 mm is reached at a time of 1245 ns when \( Qdot \) has dropped to \( 1.9 \times 10^{10} \) W. At this time, \( I_{pinch} \) is 520 kA. As \( Qdot \) continues decreasing, \( r_p \) begins to expand until it reaches a final value of 1.96 mm at 1325 ns. As the pinch expands, \( I_{pinch} \) increases slightly to 530 kA. At the end of pinch, the value of \( Q \) is 23.9 kJ or 6.8% of the bank energy \( E_0 \).

**Fig. 4.** (a) Computed total current of PF1000 at 23 kV, 1 torr Ne. The right-pointing arrow shows the start of pinch and the left-pointing arrow shows the end of pinch. (b) Computed radial dynamics of PF1000 at 23 kV, 1 torr Ne.

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**Fig. 5.** (a) Computed total current of PF1000 at 23 kV, 0.5 torr Ar. The right-pointing arrow shows the start of pinch and the left-pointing arrow shows the end of pinch. (b) Computed radial dynamics of PF1000 at 23 kV, 0.5 torr Ar. (c) Computed time histories of discharge current \( I \) (middle trace), net power emission \( \frac{dQ}{dt} (Qdot) \) (bottom trace), and piston path (\( r_p \)) (top trace): \( I \) is normalized to \( I_{peak} \) of 1970 kA, \( Qdot \) to the peak of \( 7.43 \times 10^{11} \) W, and \( r_p \) to the piston radius at the start of pinch, 14.8 mm. Time 0 is the start of radial phase. The start of pinch is at 1120 ns (where \( Qdot \) starts its vertically downward plunge). PF1000 at 23 kV, 0.5 torr Ar.

**F. PF1000 With Kr: 23 kV, 0.3 torr**

The current and radial trajectories for operation in Kr show similar features as that of Ar as shown in Fig. 5(a) and (b).

Fig. 6(a) presents some details of the last 300 ns of the radial phase, which includes the whole pinch phase. At the start of the pinch, \( r_p \) is 16.5 mm at 1256 ns, dropping sharply to 2.34 mm at 1263 ns and then further dropping less sharply to a minimum value of 0.75 mm at 1275 ns, while over this time period, the pinch current drops from 848 to 517 kA. The value of \( Qdot \) rises from \( 1.4 \times 10^{12} \) W at the start of pinch to the peak value of \( 1.2 \times 10^{13} \) W at 1261 ns and then drops sharply as plasma self-absorption, which has been rising rapidly, causes the transition from volumetric emission to surface emission at 1261 ns (computed but not shown). The value of \( Q \) at 1263 ns is 23.3 kJ (6.6% of \( E_0 \)) and at the end of pinch is 31.8 kJ (9% of \( E_0 \)).

**G. PF1000 With Xe: 23 kV, 0.2 torr**

The traces are presented in magnified scale in Fig. 7(a) to show the details of the 50 ns, which include the radiative phase up to maximum compression and a little beyond. At the start of the pinch \( r_p \) is 16.1 mm at 1272 ns, dropping sharply to 1.43 mm at 1274 ns and then further dropping less sharply to a minimum radius of 0.388 mm at 1278 ns. Over this time period, the pinch current drops...
Table III

<p>| Summary of the Numerical Experiments of PF1000 Radiative Collapse in Various Gases |
|-----------------------------------------------|----------------|----------------|-----------------|----------------|----------------|----------------|</p>
<table>
<thead>
<tr>
<th>Gas</th>
<th>$I_{peak}$ (kA)</th>
<th>$I_p$ (kA)</th>
<th>$t_{min}$ (ns)</th>
<th>$r_p$ (mm)</th>
<th>$E_{rad}$ (10$^7$ W)</th>
<th>$-Q$ (% $E_0$)</th>
<th>$-E_{rad}$ (% $E_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>853</td>
<td>789</td>
<td>206</td>
<td>23.8</td>
<td>0.192</td>
<td>-0.0005</td>
<td>-0.000002</td>
</tr>
<tr>
<td>He</td>
<td>833</td>
<td>768</td>
<td>190</td>
<td>21.9</td>
<td>0.178</td>
<td>0.0008</td>
<td>0.00004</td>
</tr>
<tr>
<td>Ne</td>
<td>819</td>
<td>650</td>
<td>222</td>
<td>15.4</td>
<td>0.063</td>
<td>0.72</td>
<td>2.63</td>
</tr>
<tr>
<td>Ar</td>
<td>820</td>
<td>530</td>
<td>208</td>
<td>14.8</td>
<td>0.016</td>
<td>7.6</td>
<td>6.8</td>
</tr>
<tr>
<td>Kr</td>
<td>848</td>
<td>307</td>
<td>206</td>
<td>16.4</td>
<td>0.007</td>
<td>116</td>
<td>9.0</td>
</tr>
<tr>
<td>Xe</td>
<td>847</td>
<td>168</td>
<td>6</td>
<td>16.1</td>
<td>0.003</td>
<td>600</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Key: $I_p$ = plasma current at start of pinch, $I_{peak}$ = plasma current at end of pinch, $t_{min}$ = time to minimum radius, $r_p$ = pinch radius at start of pinch, $E_{rad}$ = energy radiated from pinch less Joule heat energy deposited in pinch; $Q$ = energy radiated for whole pinch duration in terms of storage energy $E_0$. Notes: 1. $E_{rad}$ is predominantly bremsstrahlung in D and He; and predominantly line radiation in all the other gases. 2. In the above operating points are: D and He 3 Torr, Ne 1 Torr, Ar 0.5 Torr, Kr 0.3 Torr and Xe 0.2 Torr.

As plasma self-absorption, which has been rising rapidly, causes the transition from volumetric emission to surface emission. The value has dropped to $4 \times 10^{12}$ W at 1275 ns and further to $2 \times 10^{11}$ W at 1278 ns. The value of $Q$ at 1278 ns is 33.4 kJ (9.5%) and at the end of pinch 1480 ns is 37.0 kJ (10.5% of $E_0$), while $r_p$ has expanded to 2.6 mm [not shown in Fig. 7(a); see Fig. 7(b)]. The resistive heat liberated in the pinch over the pinch period is 43.6 kJ, while 80.6 kJ (22.9% of $E_0$) of radiation is emitted from the pinch over the period of the pinch, one-third of this amount is within the first 3 ns. Moreover, the cooling of the pinch plasma is confirmed by the temperature versus time data produced by the code. For example, in this shot for Xe over the duration of the radiation pulse, the plasma temperature dropped from $5 \times 10^6$ to $2 \times 10^6$ K.

The data for the various gases pertaining to radiative collapsibility are summarized in Table III.

In Table III, the values of peak $-Q_{dot}$, $-Q$, and $-E_{rad}$ are shown in the last three columns. The positive values in the $-Q_{dot}$ and $-Q$ columns indicate that the radiation exceeds the Joule heating in the pinch so that the net power works in the direction of radiation cooling and collapse. The values of these two quantities for D are negative, indicating that Joule heating exceeds radiation. For all the other gases, these terms act to radiatively cool the pinch although in the case of He the power is so small and the heat loss is such a small percentage of bank energy (also of pinch energy) that the effect is almost negligible and although the combined effect of SHR and $dQ/dt$ in the He pinch does show a perceptible increased compression of the pinch with a pinch radius ratio $k_{min}$ of 0.178 compared with that of D of 0.192. In Ne, the radiative cooling is unmistakable with $k_{min}$ of 0.063. Argon with $k_{min}$ of 0.016 shows a time $t_{min}$ of 130 ns to the minimum radius and then a small expansion over the rest of the pinch period. In Kr, $t_{min}$ is only 20 ns to a $k_{min}$ of 0.007. In Xe, $t_{min}$ reduces further to 6 ns with $k_{min}$ = 0.003 and a pinch radius of 0.35 mm, and in the rest of the pinch duration, over some 203 ns, the pinch expands back to almost 2 mm.

From 843 to 483 kA. The value of $Q_{dot}$ rises from $3.1 \times 10^{12}$ W at the start of pinch to the peak value of $6.0 \times 10^{13}$ W at 1274 ns and then drops sharply as plasma self-absorption, which has been rising rapidly, causes the transition from volumetric emission to surface emission. The value has dropped to $4 \times 10^{12}$ W at 1275 ns and further to $2 \times 10^{11}$ W at 1278 ns. The value of $Q$ at 1278 ns is 33.4 kJ (9.5%) and at the end of pinch 1480 ns is 37.0 kJ (10.5% of $E_0$), while $r_p$ has expanded to 2.6 mm [not shown in Fig. 7(a); see Fig. 7(b)]. The resistive heat liberated in the pinch over the pinch period is 43.6 kJ, while 80.6 kJ (22.9% of $E_0$) of radiation is emitted from the pinch over the period of the pinch, one-third of this amount is within the first 3 ns. Moreover, the cooling of the pinch plasma is confirmed by the temperature versus time data produced by the code. For example, in this shot for Xe over the duration of the radiation pulse, the plasma temperature dropped from $5 \times 10^6$ to $2 \times 10^6$ K.

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IV. CONCLUSION

In this paper, we have derived radiation levels from PF operations in a range of gases. From these and the indicative values of the reduced P-B currents for various gases from D to Xe, we have estimated the characteristic depletion times of pinch energy due to radiation minus Joule heating. These depletion times indicate that D and He will have little or no radiation cooling, that Ne will show significant radiation cooling leading to substantial radiative compression, and that Ar, Kr, and Xe will have severe radiative collapse. These results (Tables I and II) are estimated without considering the plasma opacity (plasma self-absorption). The numerical experiments, which include self-absorption, demonstrate the substantial moderating effects of self-absorption and nevertheless confirm the indications of Tables I and II. The exceptionally small radii reached by these radiation-enhanced compressions could be useful in situations where small-sized radiation sources are needed.

We note that the code assumes that the pinch is compressed as a column. In the actual operation, break up of the column into a line of spots has been observed particularly, but not exclusively in the heavier gases [64]. Such breakups may lead to localized enhanced compression and may tend to make it easier for radiative collapse to occur. Moreover, the action of beams will also remove energy from the pinch [49], [50]. If beams are emitted even partially within the pinch time, this could also lead to beam-enhanced radiative collapse.

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REFERENCES


Authors’ photographs and biographies not available at the time of publication.