

# IPFS

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## Institute for Plasma Focus Studies

### Internet Workshop on Numerical Plasma Focus Experiments

(Supplementary Notes SP3 for Module 3)

[in part extracted from [file 2Theory.pdf](#) from:

<http://www.intimal.edu.my/school/fas/UFLF/> ] and from various papers

#### Radiation Terms

The Bremsstrahlung loss term may be written as:

$$\frac{dQ_B}{dt} = -1.6 \times 10^{-40} N_i^2 (\pi r_p^2)_{z_f} T^{1/2} z^3$$

$$N_o = 6 \times 10^{26} \frac{\rho_o}{M}; \quad N_i = N_o f_{mr} \left( \frac{a}{r_p} \right)^2$$

Recombination loss term is written as:

$$\frac{dQ_{rec}}{dt} = -5.92 \times 10^{-35} N_i^2 Z^5 (\pi r_p^2)_{z_f} / T^{0.5}$$

The line loss term is written as:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} N_i^2 Z Z_n^4 (\pi r_p^2)_{z_f} / T$$

$$\text{and } \frac{dQ}{dt} = \frac{dQ_J}{dt} + \frac{dQ_B}{dt} + \frac{dQ_L}{dt} + \frac{dQ_{rec}}{dt}$$

where  $dQ/dt$  is the total power gain/loss of the plasma column.

By this coupling, if, for example, the radiation loss  $\left( \frac{dQ_B}{dt} + \frac{dQ_L}{dt} \right)$  is severe, this would

lead to a large value of  $\frac{dr_p}{dt}$  inwards. In the extreme case, this leads to radiation collapse, with  $r_p$  going rapidly to zero, or to such small values that the plasma becomes opaque to the outgoing radiation, thus stopping the radiation loss.

This radiation collapse occurs at a critical current of 1.6 MA (the Pease-Braginski current) for deuterium. For gases such as Neon or Argon, because of intense line radiation, the critical current is reduced to even below 100kA, depending on the plasma temperature.

### Plasma Self Absorption and transition from volumetric emission to surface emission

Plasma self absorption and volumetric (emission described above) to surface emission of the pinch column have been implemented in the following manner.

The photonic excitation number (see [File 3 Appendix](#) by N A D Khattak) is written as follows:

$$M = 1.66 \times 10^{-15} r_p Z_n^{0.5} n_i / (Z T^{1.5}) \text{ with } T \text{ in eV, rest in SI units}$$

The volumetric plasma self-absorption correction factor A is obtained in the following manner:

$$A_1 = (1 + 10^{-14} n_i Z) / (T^{3.5})$$

$$A_2 = 1 / AB_1$$

$$A = A_2^{(1+M)}$$

Transition from volumetric to surface emission occurs when the absorption correction factor goes from 1 (no absorption) down to 1/e (e=2.718) when the emission becomes surface-like given by the expression:

$$\frac{dQ}{dt} = -const x Z_n^{3.5} Z^{0.5} \left( r_p \right) Z_f T^4$$

where the constant *const* is taken as  $4.62 \times 10^{-16}$  to conform with numerical experimental observations that this value enables the smoothest transition, in general, in terms of power values from volumetric to surface emission.

Where necessary another fine adjustment is made at the transition point adjusting the constant so that the surface emission power becomes the same value as the absorption corrected volumetric emission power at the transition point. Beyond the transition point (with A less than 1/e) radiation emission power is taken to be the surface emission power.

### Neutron Yield

<http://www.intimal.edu.my/school/fas/UFLF/>

Adapted from the following papers (with modifications for erratum)

**Pinch current limitation effect in plasma focus** (This version includes an Erratum)

**S. Lee and S. H. Saw, *Appl. Phys. Lett.* 92, 021503 (2008), DOI:10.1063/1.2827579**

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<http://link.aip.org/link/?APPLAB/92/021503/1>

**Neutron Scaling Laws from Numerical Experiments** (This version includes an Erratum)

**S Lee and S H Saw, *J of Fusion Energy*, DOI: 10.1007/s10894-008-9132-7**

published first online 20 February 2008 at <http://dx.doi.org/10.1007/s10894-008-9132-7>

"The original publication is available at [www.springerlink.com](http://www.springerlink.com)."

Neutron yield is calculated with two components, thermonuclear term and beam-target term.

The thermonuclear term is taken as:

$$dY_{th} = 0.5n_i^2 (3.142) r_p^2 z_f \langle \sigma v \rangle (\text{time interval})$$

where  $\langle \sigma v \rangle$  is the thermalised fusion cross section-velocity product corresponding to the plasma temperature, for the time interval under consideration. The yield  $Y_{th}$  is obtained by summing up over all intervals during the focus pinch.

The beam-target term is derived using the following phenomenological beam-target neutron generating mechanism<sup>17</sup>, incorporated in the present RADPFV5.13. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. In this modeling each factor contributing to the yield is estimated as a proportional quantity and the yield is obtained as an expression with proportionality constant. The yield is then calibrated against a known experimental point.

$$\text{The beam-target yield is written in the form: } Y_{b-t} \sim n_b n_i (r_p^2 z_p) (\sigma v_b) \tau$$

where  $n_b$  is the number of beam ions per unit plasma volume,  $n_i$  is the ion density,  $r_p$  is the radius of the plasma pinch with length  $z_p$ ,  $\sigma$  the cross-section of the D-D fusion reaction, n- branch<sup>18</sup>,  $v_b$  the beam ion speed and  $\tau$  is the beam-target interaction time assumed proportional to the confinement time of the plasma column.

Total beam energy is estimated<sup>17</sup> as proportional to  $L_p I_{pinch}^2$ , a measure of the pinch inductance energy,  $L_p$  being the focus pinch inductance. Thus the number of beam ions is  $N_b \sim L_p I_{pinch}^2 / v_b^2$  and  $n_b$  is  $N_b$  divided by the focus pinch volume. Note that  $L_p \sim \ln(b/r_p) z_p$ , that<sup>4</sup>  $\tau \sim r_p \sim z_p$ , and that  $v_b \sim U^{1/2}$  where  $U$  is the disruption-caused diode voltage<sup>17</sup>. Here 'b' is the cathode radius. We also assume reasonably that  $U$  is proportional to  $V_{max}$ , the maximum voltage induced by the current sheet collapsing radially towards the axis.

$$\text{Hence we derive: } Y_{b-t} = C_n n_i I_{pinch}^2 z_p^2 ((\ln b/r_p)) \sigma / V_{max}^{1/2} \quad (1)$$

where  $I_{pinch}$  is the current flowing through the pinch at start of the slow compression phase;  $r_p$  and  $z_p$  are the pinch dimensions at end of that phase. Here  $C_n$  is a constant which in practice we will calibrate with an experimental point.

The D-D cross-section is highly sensitive to the beam energy so it is necessary to use the appropriate range of beam energy to compute  $\sigma$ . The code computes  $V_{max}$  of the order of 20-50 kV. However it is known<sup>17</sup>, from experiments that the ion energy responsible for the beam-target neutrons is in the range 50-150keV<sup>17</sup>, and for smaller lower-voltage machines the relevant energy<sup>19</sup> could be lower at 30-60keV. Thus to align with experimental observations the D-D cross section  $\sigma$  is reasonably obtained by using beam energy equal to 3 times  $V_{max}$ .

A plot of experimentally measured neutron yield  $Y_n$  vs  $I_{pinch}$  was made combining all available experimental data<sup>2,4,12,13,17,19-22</sup>. This gave a fit of  $Y_n = 9 \times 10^{10} I_{pinch}^{3.8}$  for  $I_{pinch}$  in the range 0.1-1MA. From this plot a calibration point was chosen at 0.5MA,  $Y_n = 7 \times 10^9$  neutrons. The model code<sup>23</sup> RADPFV5.13 was thus calibrated to compute  $Y_{b-t}$  which in our model is the same as  $Y_n$ .

## Notes on **The total current and $I_{\text{peak}}$ , the plasma current and $I_{\text{pinch}}$**

Extracted From: [Computing Plasma Focus Pinch Current from Total Current Measurement](#)  
S. Lee, S. H. Saw, P. C. K. Lee, R. S. Rawat and H. Schmidt, *Appl Phys Letters* **92**, 111501  
(2008) DOI:10.1063/1.2899632

The total current  $I_{\text{total}}$  waveform in a plasma focus discharge is easily measured using a Rogowski coil. The peak value  $I_{\text{peak}}$  of this trace is commonly taken as a measure of the drive efficacy and is often used to scale the yield performance of the plasma focus. This is despite the fact that yields should more consistently be scaled to focus pinch current  $I_{\text{pinch}}$ , since it is  $I_{\text{pinch}}$  which directly powers the emission processes. The reason many researchers use  $I_{\text{peak}}$  instead of  $I_{\text{pinch}}$  for scaling is simply that while  $I_{\text{peak}}$  is easily measured,  $I_{\text{pinch}}$ , which is the value of the plasma sheath current  $I_p$  at time of pinch, is very difficult to measure even in large devices where it is possible to place magnetic probes near the pinch. This measurement is also inaccurate and perturbs the pinch. In a small device, there is no space for such a measurement.

The relationship between  $I_{\text{pinch}}$  and  $I_{\text{peak}}$  is not simple and has only been recently elaborated. It primarily depends on the value of the static inductance  $L_0$  compared to the dynamic inductances of the plasma focus. As  $L_0$  is reduced, the ratio  $I_{\text{pinch}} / I_{\text{peak}}$  drops. Thus, yield laws scaled to  $I_{\text{peak}}$  will not consistently apply when comparing two devices with all parameters equal but differing significantly in  $L_0$ . Better consistency is achieved when yield laws are scaled to  $I_{\text{pinch}}$ . In this paper, we propose a numerical method to consistently

### **Distinguishing the $I_{\text{total}}$ waveform from the $I_p$ waveform**

A measured trace of  $I_{\text{total}}$  is commonly obtained with a Rogowski coil wrapped around the plasma focus flange through which is fed  $I_{\text{total}}$  discharged from the capacitor bank between the coaxial electrodes across the back wall. A part of  $I_{\text{total}}$ , being the plasma sheath current  $I_p$ , lifts off the back-wall insulator and drives a shock wave axially down the coaxial space. We denote  $f_c$  as the current fraction  $I_p / I_{\text{total}}$  for the axial phase and  $f_{cr}$  for the radial phases. In modeling it is found that a reasonable value for initial trial for  $f_c$  is 0.7 with a similar first trial value for  $f_{cr}$ . However in a DPF78 experiment  $f_c$  was found to vary from 0 at the start of the axial phase rising rapidly above 0.6 for the rest of the axial phase. In the radial phase  $f_{cr}$  was found to stay above 0.6 before dropping to 0.48 at the start of the pinch and then towards 0.4 as the pinch phase progressed. These Stuttgart results confirm a complex relationship between the waveforms of  $I_{\text{total}}$  and  $I_p$ .

The performance of a plasma focus is closely linked to the current  $I_{\text{pinch}}$  actually participating in the focus pinch phase rather than the total current flowing in the circuit. It is a common practice to take  $I_{\text{peak}}$  or some representative fraction of it as  $I_{\text{pinch}}$ . Another practice is to take the value of  $I_{\text{total}}$  at the time of the pinch as  $I_{\text{pinch}}$ . Whilst in their special cases this practice could be justifiable, the distinction of  $I_p$  from  $I_{\text{total}}$  should generally be clearly made. We emphasize that it should be the value of  $I_p$  at the time of pinch which is the relevant value for the purpose of yield scaling. The practice of associating yield

scaling with the total current waveform (i.e. taking  $I_{\text{peak}}$  or  $I_{\text{total}}$  at estimated pinch time) would be justifiable if there were a linear relationship between the waveforms of  $I_{\text{total}}$  and  $I_p$ . However as shown by the Stuttgart experiments the actual relationship is a very complex one which we may ascribe to the interplay of the various electro-dynamical processes including the relative values of static inductance  $L_o$ , tube inductance and the dynamic resistances which depend on the tube geometry and plasma sheath speeds. This relationship may change from one machine to the next. Whilst these electro-dynamical processes and other relevant ones such as radiation are amenable to modeling there are other machine effects such as back wall restriking (for example due to high induced voltages during the pinch phase) which can almost unpredictably affect the relationship between  $I_{\text{total}}$  and  $I_p$  during the crucial radial phases. Hence it is not only simplistic to discuss scaling in terms of the  $I_{\text{total}}$  waveform (i.e. taking  $I_{\text{peak}}$  or the value of  $I_{\text{total}}$  at the estimated time of pinch) but also inconsistent. One of the most important features of a plasma focus is its neutron production. The well-known neutron yield scaling, with respect to current, based on various compilations of experimental data, is  $Y_n \sim I_{\text{pinch}}^x$  where  $x$  is varied in the range 3–5. In a recent paper, numerical experiments using a code was used to derive a scaling with  $x = 4.7$ . Difficulties in the interpretation of experimental data ranging across big and small plasma focus devices include the assignment of the representative neutron yield  $Y_n$  for any specific machine and the assignment of the value of  $I_{\text{pinch}}$ . In a few larger machines attempts were made to measure  $I_{\text{pinch}}$  using magnetic probes placed near the pinch region, with uncertainties of 20%. Moreover the probes would have affected the pinching processes. In most other cases related to yield scaling data compilation or interpretation  $I_{\text{pinch}}$  is simply assigned a value based on the measurement of peak total current  $I_{\text{peak}}$  or the value of total current at the observed current dip.

The difficulties in distinguishing  $I_{\text{pinch}}$  from  $I_{\text{total}}$  are obviated in numerical experiments using the Lee Model [In a typical simulation, the  $I_{\text{total}}$  trace is computed and fitted to a measured  $I_{\text{total}}$  trace from the particular focus. Three model parameters for fitting are used: axial mass swept-up factor  $f_m$ , current factor  $f_c$  and radial mass factor  $f_{mr}$ . A fourth model parameter, radial current factor,  $f_{cr}$  may also be used. When correctly fitted the computed  $I_{\text{total}}$  trace agrees with the measured  $I$  trace in peak amplitude, rising slope profile and topping profile which characterize the axial phase electro-dynamics. The radial phase characteristics are reflected in the roll-over of the current trace from the flattened top region, and the subsequent current drop or dip. Any machine effects, such as restrikes, current sheath leakage and consequential incomplete mass swept up, not included in the simulation physics is taken care of by the final choice of the model parameters, which are fine-tuned in the feature-by-feature comparison of the computed  $I_{\text{total}}$  trace with the measured  $I_{\text{total}}$  trace. Then there is confidence that the computed gross dynamics, temperature, density, radiation, plasma sheath currents, pinch current and neutron yield may also be realistically compared with experimental values.

#### A note on scaling:

Scaling of yields to say  $I_{\text{pinch}}$  should be carried out using yields which are at optimum, or at least near optimum. If one indiscriminately uses any data one may end up with

completely trivial or misleading results. For example if a point is used at too high or low pressure (away from the optimum pressure) then there may be zero yield ascribed to values of  $I_{\text{pinch}}$ .

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<http://www.intimal.edu.my/school/fas/UFLF/>

**Reference to this course and the Lee model code should be given according to the following format:**

**Lee S. Radiative Dense Plasma Focus Computation Package (2008): RADPF**  
[www.plasmafocus.net](http://www.plasmafocus.net)    [www.intimal.edu.my/school/fas/UFLF/](http://www.intimal.edu.my/school/fas/UFLF/)