Dimensions and Lifetime of the Plasma Focus Pinch

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Abstract— **In spite of intensive studies of the plasma focus, scaling laws, with the exception of those for neutron yield, are not well known. This paper points out that Mather-type plasma focus devices operated in the neutron-optimized regime have remarkably little variation in a drive parameter, $(I_p/a)/\rho_o^{1/2}$, the peak drive current per unit anode radius divided by the square root of the fill density; this quantity having the value of 89 kA/cm per torr deuterium equivalent of fill gas with a standard deviation of less than 10%. This parameter controls the speed of the plasma in both the axial and radial phases; and its constancy over a wide range of plasma focus devices really indicates that these devices all operate at the same axial and radial speeds, and hence by inference they all have the same temperatures in the axial and radial phases. Using a simple dynamical model the linear dimensions and time scales of the gross plasma focus pinch are shown to be related to the anode radius of the plasma focus device. The results of experiments performed with a 3-kJ device using different anodes are in good agreement with the theoretical predictions.**

I. Introduction

The PLASMA focus is an important device for the generation of intense radiation including neutrons, X-rays, and particle beams. The physics underlying the mechanisms for the generation of these radiations in the plasma focus is not known, although there has been intensive investigations for the past three decades. Experimental and theoretical work on the focus has reached quite high levels. For example, detailed simulation work on the plasma focus had been carried out since 1971 [1] and a large range of devices has been constructed from 100 J small focus to 1 MJ large focus. Advanced experiments have been carried out on the dynamics, radiation, instabilities, and nonlinear phenomena. Yet despite all these intensive studies, very little regarding scaling appears to be known or at least documented, with the exception of the scaling law for neutron yield. In order to acquire a better overall understanding of the plasma focus it may be useful to start by looking at the scaling of simple quantities. For example, it needs to be pointed out that neutron-optimized Mather-type plasma focus, from small to large devices, all have the same drive parameter $(I_p/a)/\rho_o^{1/2}$, where I_p is the peak current driving the plasma sheath, a the anode radius, and ρ_o is the ambient gas density. Strictly speaking, the drive parameter should include a mass sweeping factor in the density as it is known that the mass swept up by the current sheath is not 100% of the mass encountered so that the "effective" gas density is actually less. Also, a current factor should be included in the drive parameter. Not including the mass and current factors in the drive parameter is equivalent to assuming that these factors remain constant over the range of machines. Data of the drive parameter are compiled in Table I. In this compilation care is taken that the data represent actual neutron-optimized shots as published. Actual-shot data need to be distinguished from machine data which may give maximum current capability of a machine or design current which when combined with the pressure of an actual shot may give a misleading drive parameter since the actual shot is usually carried out not at the maximum or designed current. Thus much published data of machines could not be used in this compilation because of uncertainty over actual shot data. In Table I the value of the peak current I_p flowing into the focus tube is used as this value is more usually quoted than the plasma current and thus using the peak current more actual shot data points are available.

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TABLE I
Actual-Shot Data of Some Neutron-Optimized Mather-Type Plasma Focus Devices and Their Calculated Drive Parameter. The Mean Value and the Corresponding Standard Deviation of the Drive Parameter Are 89.0 and 7.7 kA/cm/(torr D_2)$^{1/2}$, Respectively

Device/Author [Reference]	V (kV)	W (kJ)	p (torr D_2)	I_p (kA)	a (cm)	$I_p/a/p^{1/2}$ (kA/cm/[torr D_2]$^{1/2}$)
DPF-78 [16]	60	28	5	550	2.5	98.4
SPEED 2 [17]	183	70	4	1000	6	83.3
PF-360 [18]	21	60	3	750	5	86.6
Mather [19]	17.3	13.5	6.5	537	2.5	84.3
UMPF [20]	14	6	6.5	300	1.25	94.1
UNU/ICTP-PFF [10,15]	14	2.9	3.5	172	0.95	96.8
Bernstein [21]		27	8	820	3.9	74.3
POSEIDON [22]	60	280	3.8	1200	6.55	94.0

Table I shows that the drive parameter for a range of neutron-optimized plasma focus devices is 89 ± 8 kA/cm per (torr)$^{1/2}$ of deuterium. The standard deviation of less than 10% is quite remarkable since the machines in the compilation range from 3 to 280 kJ and include early machines of Mather and Bernstein, advanced machines such as SPEED 2, and small training machines.

Any reasonable modeling will show that the drive parameter determines the speed in both the axial and radial phases [2], [3]. This will be shown in the following section. Having the same drive parameter indicates that the range of machines surveyed all operate with the same axial and radial speeds. This is not contradictory to the often reported speeds of just under 10 cm/μs for the axial phase just before the radial phase and a radial speed of about 25 cm/μs when the imploding shock is nearing the axis. The on-axis radial shock speed is seldom given as it is difficult to determine experimentally. Since the energetics of the plasma focus is determined by the shock speed up to the time the radial shock goes on axis, constancy of the drive parameter gives a strong indication that the temperature is also the same over the range of plasma focus surveyed. The density also varies remarkably little, a range of two, considering the large range of energies encompassed in the survey. Thus the plasma energy density of the machines has also relatively little variation. The mechanisms for radiation of energy and particles depend ultimately on the energy density and/or the temperature. The small variations observed in these quantities over the large range of machines while on the one hand is an advantage for comparing experiments may perhaps on the other hand act as a limitation. For example, in terms of fusion neutrons, the thermonuclear fusion cross section scales as temperature $T^{4.5}$ in the region of 1 keV which is the temperature regime of the plasma focus. Would it not be an advantage to increase the drive parameter while still maintaining the strong focusing characteristics since the shocked plasma temperature scales as the square of the drive parameter [2]? By increasing the value of the drive parameter, the shocked plasma temperature is increased, and hence the thermonuclear yield. The scaling in the limit can be $Y \sim$ (drive parameter) to the power of nine.

The above discussion centers around the drive parameter and its impact on plasma focus energetics. The identification of the constancy of the drive parameter as a possible limitation on focus performance may be useful in considerations of scaling. Other factors such as geometrical and temporal characteristics may also be useful for scaling of radiation performance; for example, the size of the focused plasma and the lifetime. Reviews in the literature [4]–[8] give a range of plasma dimensions of 1–15 mm in the diameter and 10–70 mm in the length and the pinch lifetime (defined as the time from the first compression to a minimum radius to the time of violent breakup of the plasma column) of tens of nanoseconds to 100–200 ns. However, no attempt has been made to discuss how these dimensions and times scale with, for example, stored energy or current. Yet by considering any simple model of the plasma focus dynamics the dimensions and time scales of the gross plasma focus pinch are easily related to the parameters of the plasma focus device.

II. Theory

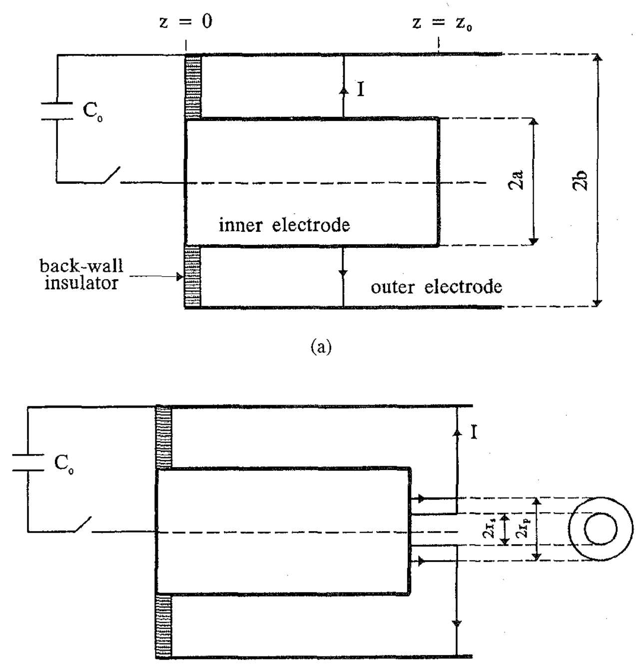
Using a snowplow model for the axial phase and a slug model for the radial phase (see Fig. 1), characteristic axial and radial transit times for the plasma shock front are, respectively, [9]–[11]

$$t_a = \left[\frac{4\pi^2(c^2-1)}{\mu \ln c}\right]^{1/2} \frac{z_o \rho_o^{1/2}}{\left(\frac{I_o}{a}\right)} \tag{1}$$

and

$$t_r = \left[\frac{16\pi^2}{\mu(\gamma+1)}\right]^{1/2} \frac{a \rho_o^{1/2}}{\left(\frac{I_o}{a}\right)} \tag{2}$$

where $c = b/a$, b is outer radius of coaxial electrodes, a is inner radius, μ is permeability, z_o is length of inner electrode, I_o is characteristic current, ρ_o is ambient density, and γ is the specific heat ratio.



(b)

Fig. 1. Schematic of the dynamics of the plasma focus for the one-dimensional computational model: (a) axial phase—snowplow model, (b) radial phase—slug model.

From these we may write the characteristic speeds for the axial and radial shock, respectively, as

$$c_a = \frac{z_o}{t_a} = \frac{\left(\frac{I_o}{a}\right)}{\rho_o^{1/2}} \left[\frac{\mu \ln c}{4\pi^2(c^2-1)}\right]^{1/2} = \frac{\left(\frac{I_o}{a}\right)}{\rho_o^{1/2}} g_a \tag{3}$$

and

$$c_r = \frac{a}{t_r} = \frac{\left(\frac{I_o}{a}\right)}{\rho_o^{1/2}} \left[\frac{\mu(\gamma+1)}{16\pi^2}\right]^{1/2} = \frac{\left(\frac{I_o}{a}\right)}{\rho_o^{1/2}} g_r \tag{4}$$

where g_a and g_r are the effective geometrical constants governing the axial and radial phases, respectively; g_r is a constant and g_a varies by less than 10% from a mean value of 0.52 over the range of $c = 1.5$ to 2 for the machines of Table I. Hence the characteristic speeds c_a and c_r depend primarily on the characteristic drive parameter $(I_o/a)/\rho_o^{1/2}$; and the operational speeds in the axial and radial phases depend only on the operational drive parameter $(I_p/a)/\rho_o^{1/2}$, where I_p is the peak current entering the focus tube. A similar parameter has been discussed earlier [12].

A radial collapse model which has proven useful for comparison with experimental work is shown in Fig. 1(b). Essentially, a zero length pinch starts at $r = a$ and elongates as it pinches inwards. The collapsing magnetic piston at position r_p pushes ahead of it a radially converging structure at position r_s. Using this model and having shown that the characteristic speed of radial transit is the same over the range of devices, it follows that the compression time $t = t_{\text{comp}}$ (see Fig. 2) depends on the radius a of the anode. As the shock converges on the axis we may define the position of the piston at this time as r_c.

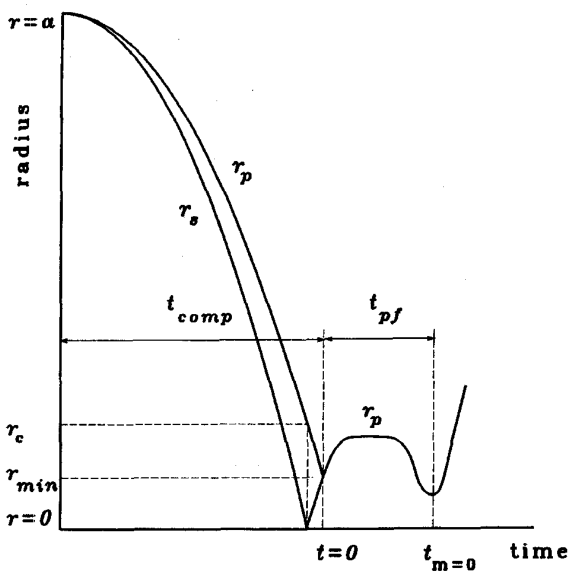


Fig. 2. Radial trajectories of shock (r_s) and piston (r_p) as a function of time.

After the shock hits the axis a reflected shock propagates outward. In the meantime the piston (see Fig. 2) continues inwards until it is hit by the reflected shock at which point the radially inward motion of the piston is reversed at position $r_p = r_{\min}$. Experimentally it is observed that the plasma column usually pinches in a second time sometimes to an even smaller radius before finally exploding outward. This ends the pinch phase and defines the quantity $t_{\rm pf}$ as shown in Fig. 2.

A numerical computation of this model [9]–[11], [13] shows that both r_c and $r_{\min}$ are proportional to the anode radius a.

The ending of the pinch phase is observed to be caused by instabilities. Thus we may relate the period $t_{\rm pf}$ to the transit time of small disturbances across the compressed plasma column. The speed of these small disturbances is practically a constant over the range of devices because of the near constancy of plasma temperature, density, and magnetic flux. Thus in such a scenario $t_{\rm pf}$ also depends on the radius a.

We have carried out a numerical computation with the endpoint fixed by an energy-balance method [14] and found the following for the deuterium focus: $r_{\min} \approx 0.12a; z_{\max} \approx 0.8a; t_{\rm comp} \approx 4.5a; t_{\rm pf} \approx 2a$, with a in millimeters, $t_{\rm comp}$ and $t_{\rm pf}$ in nanoseconds. Here, $z_{\max}$ is the length of the focus column measured from the face of the anode.

Similar results are applicable to the gross pinch of the plasma focus in other gases such as Argon, Neon, and Xenon. Generally for these other gases $r_{\min} \leq 0.1a$. The difference in the computation involves the value of γ. Scaling these parameters of the focus to the anode radius a is extremely useful for understanding and for applications. We have carried out experiments to confirm these theoretical results.

III. Experiments

Experiments were carried out using a 3 kJ plasma focus device [15] operated at 14 kV in deuterium at the neutron optimized pressure of 4.5 mbar. The inner electrode was the anode. All anodes tested were made of copper and hollow at the open end.

TABLE II
Mean Values of the Pinch Lifetime ($t_{\rm pf}$) and of the Compression Time ($t_{\rm comp}$) as Functions of the Anode Radius (a)

a (mm)	t_{comp} (ns)	t_{pf} (ns)
9.5	45.1	19.5
8.75	41.1	17.6
8.5	39.2	17.1
7.5	31.4	15.2
7	29.1	14.3

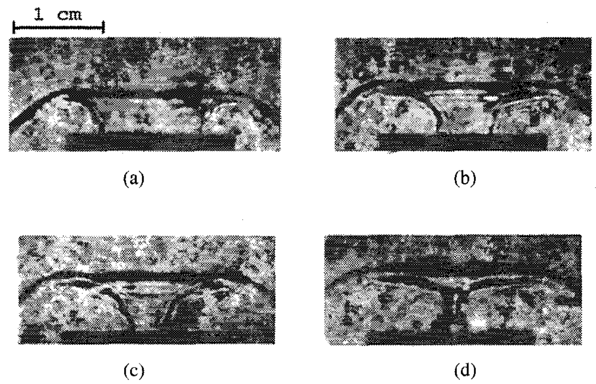


Fig. 3. Examples of shadowgraphs of plasma focus during the radial phase for an anode of 9.5 mm radius.

The diagnostics employed included [15]: shadowgraphy using a TEA nitrogen laser, current (I) and current derivative (dI/dt) using current transformer and Rogowski coil, respectively, voltage (V) using a resistive voltage probe, soft X-ray emission rate ($dY_{\rm SX}/dt$) using filtered PIN diodes, and hard X-ray and neutron time-resolved measurements ($dY_{\rm n}/dt$ and $dY_{\rm HX}/dt$, respectively) using two fast response plastic scintillator-photomultiplier systems in a time-of-flight arrangement. The field of view of the shadowgraph technique is above the open end of the anode. The X-ray and neutron emission rates were recorded in a radial plane with respect to the axis of the focus tube.

The dimensions of the pinch were determined from the time sequences of shadowgraphs. The timing of the shadowgraphs as well as the timing of the important periods of the evolution of the plasma focus were performed with better than 1 ns accuracy. The time reference was taken at the instant when the plasma column reached the minimum radius ($t = 0, r = r_{\min}$). The instant $t = 0$ was identified from the current derivative signal (the first sharp negative spike) and from the soft X-ray signal (the first spike). The pinch lifetime ($t_{\rm pf}$) was defined as the duration from $t = 0$ to the instant corresponding to the development of the $m = 0$ instability ($t_m = 0$). The latter is associated with the beginning of the hard X-ray pulse.

Experiments were performed with several anodes of radii of 9.5, 8.75, 8.5, 7.5, and 7 mm, respectively [15]. For each electrode configuration a large number of discharges (more than 50) was analyzed. Our statistical analysis was performed

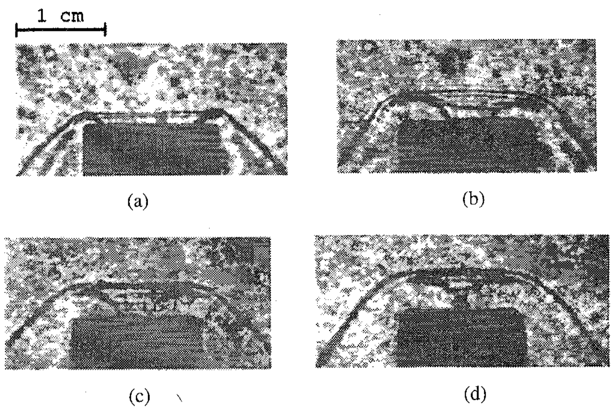


Fig. 4. Examples of shadowgraphs of plasma focus during the radial phase for an anode of 7.5 mm radius.

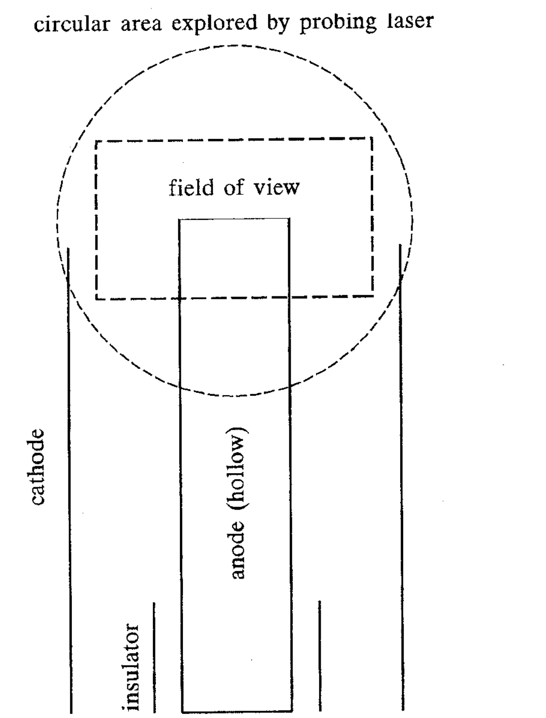


Fig. 5. Field of view of shadowgraphs at the open end of the electrode system.

on series of shots without excluding the discharges with very low neutron yield. Detailed discussions and correlations of the discharge parameters (e.g., peak current, peak axial velocity, axial transit time, radial compression time, and pinch lifetime) and neutron and X-ray emission were presented elsewhere [15]. We would like to mention only that the standard deviation of the peak discharge current was less than 2%, the standard deviation of the peak axial speed was less than 7% while the standard deviation of the neutron yield reached 60% for some anode configurations. The mean values of the pinch lifetime and of the compression time are illustrated in Table II. They show that the pinch lifetime (t_{pf}) as well as the compression time (t_{comp}) are scaled to the anode radius in agreement with the theoretical predictions.

The evolution of the plasma during the radial compression phase and pinch phase was analyzed from shadowgraphs. Some images of the plasma during the final stages of the compression phase and pinch phase are illustrated in Figs. 3 and 4 for two anodes of radii 9.5 mm and 7.5 mm, respectively. The field of view of the shadowgraphs is illustrated in Fig. 5. From the shadowgraphs, r_p is estimated with 10% accuracy.

Measurements of the axial elongation and radius of the plasma column during the radial collapse phase are in good agreement with the theoretical model. At the end of the radial implosion the following experimental values were determined:

when $a = 9.5$ mm it was found that:

$$\begin{cases} r_{\min} \approx 1.2 \text{ mm} \approx 0.13a \\ z_{\max} \approx 7.5 \text{ mm} \approx 0.8a \end{cases}$$

and

when $a = 7.5$ mm it was found that:

$$\begin{cases} r_{\min} \approx 1 \text{ mm} \approx 0.13a \\ z_{\max} \approx 6 \text{ mm} \approx 0.8a. \end{cases}$$

Here, $z_{\max}$ is the length of the focus column measured from the face of the anode. These results confirm our theoretical predictions.

IV. Conclusion

Numerical computations on a simple model yield the following dimensions and times for the gross pinch phase of deuterium plasma focus:

$$r_{\min} \approx 0.12a \quad z_{\max} \approx 0.8a$$
$$t_{\mathrm{comp}} \approx 4.5a \qquad t_{\mathrm{pf}} \approx 2a \quad (t \text{ in ns, } a \text{ in mm}).$$

Experimental results are presented which verify the predictions of the model.

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